# A Theory of Pre-litigation Settlement and Patent Assertion Entities

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**Abstract**

An informed (potential) plaintiff can demand a payment from an uninformed (potential) defendant with the threat of a law suit that may not be credible. The communication takes place before the law suit is filed and is costless to both sides. A unique equilibrium that satisfies the intuitive criterion exists and exhibits partial pooling—cases below a cutoff of legal damage reach pre-litigation settlement, while those above are filed for suit. This equilibrium offers an explanation for the distinctive behaviors of litigation PAEs who aggressively file law suits for settlements and portfolio PAEs who obtain licensing revenues through pre-litigation settlements. Among other results, we shows that, in the US legal system, increasing a plaintiff's legal costs or adopting the fee-shifting rule reduces the number of filed law suits while risking driving more cases toward pre-litigation settlements. Surprisingly, reducing a defendant's legal costs or increasing trial costs has the opposite effects.

# 1 Introduction

Individuals and entities resolve countless disputes in the shadow of litigation. Most of these resolutions involve a settlement even before the case is formally filed for law suit—a fact beckoning a pair of intriguing questions. On the one hand, if cases can be resolved through pre-litigation settlement without incurring any legal cost, why are they ever filed for law suit? On the other hand, if the litigants have to expend extraordinary resources to reach a settlement post-litigation, why did they fail to resolve the dispute earlier?

Patent assertion entities (PAEs) or "patent trolls," who derive most of their revenues by asserting patent rights against alleged infringers instead of applying patents for productive uses, are a case in point. Their entire business model hinges on legal settlements, which are frequently called *ex-post* licensing fees. Two distinct practices stand out among PAEs in a case study of the wireless chipset industry by the Federal Trade Commission (2016). Some PAEs predominantly send demand letters with litigation threats to extract pre-litigation settlements, or so-called licensing fees, from other producing entities. Others file many law suits directly, and reach *ex-post* licensing settlements quickly after filing. Most surprising is the fact that the former type, which may appear more "troll-like," owns much larger patent pools, files occasional suits, settles much later in the legal process, and receives much higher pre-trial (or post-filing) settlements. On the contrary, the latter type accounts for the majority of cases filed for suit and, surprisingly, derives much less total revenue and smaller pre-trial settlements. Indeed, Federal Trade Commission (2016) refers to the former type as "portfolio PAEs" and the latter as "litigation PAEs."[1] If portfolio PAEs can extract settlements before litigation, why bother filing suits at all? Moreover, why do the litigation PAEs not imitate the practices of the portfolio PAEs to file fewer suits and extract more settlements? A framework analyzing policy implications related to PAE activities must pass a test by explaining these observations consistently.

This paper presents a model of pre-litigation settlement. A risk-neutral plaintiff sustains some privately observed legal damage from a (potential) defendant. The risk neutral defendant has a commonly known belief over the distribution of the damage, but not its exact value. Before formally filing a law suit, the plaintiff may choose to demand a settlement payment from the defendant in exchange for release from further litigation. If the defendant pays a desirable settlement, the dispute is resolved. Otherwise, if the case has sufficient merit, the plaintiff may choose to file suit, which incurs further legal costs for both parties. After the suit is filed and some legal costs incurred, the defendant learns the true damage through discovery. Then, both sides may have another opportunity to settle before going to trial for the court decision.

The model explores a context in which an informed plaintiff makes a demand to an uninformed defendant, while (1) the threat of a law suit may not be credible; (2) the right to file suit resides with the informed plaintiff; and (3) entry to the settlement bargaining game is free. These three features are defining characteristics in contexts involving litigation threats such as NPE-related law suits. The second feature sets the current model apart from previous works concerning *non-credible* cases or *negative expected value (NEV)* cases, whose expected court judgment is dwarfed by the legal costs involved in pursuing litigation, such that they would be automatically dropped before proceeding to the next stage of the litigation process (Nalebuff, 1987; Bebchuk, 1988; Meurer, 1989). The third feature highlights the distinction between the pre-litigation settlement context and the well-studied pre-trial bargaining context, in which the plaintiff has to pay an initial litigation cost to enter the bargaining game (Reinganum and Wilde, 1986; Farmer and Pecorino, 2007).

We characterize all equilibria with pre-litigation settlement under continuous and differentiable probabil-

[1] Lemley and Melamed (2013) applies three categories for NPEs, but with consistent properties.

ity distributions over the legal damage. In particular, among all equilibria, there is a unique equilibrium that satisfies the intuitive criterion (Cho and Kreps, 1987), which is also the equilibrium that maximizes the plaintiff's pre-litigation settlement and minimizes legal costs. All equilibria with pre-litigation settlements exhibit a partial pooling structure, in which plaintiffs with case values below a cutoff send out identical demands, inducing the defendants to pay a pre-litigation settlement, while plaintiffs with case values above the cutoff file law suits.

In these equilibria, all defendants with NEV cases receive a demand, but not all receiving a demand face NEV cases. With the correct belief of the plaintiff's strategy, the defendant forms a correct belief about the chance that the case will actually be litigated if he or she defaults on the received demand. The defendant calculates the expected loss by weighing potential legal damages plus legal costs with the chance that the plaintiff will file the case, and is therefore willing to pay this amount pre-litigation to avoid a law suit. However, because the defendant's highest willingness to pay before litigation is weighed down by low value cases and the possibility of empty threats, plaintiffs with high enough case values would rather incur extra legal costs to litigate and force the defendant to discover the case value. Once the high case value is credibly revealed to the defendant through discovery, the pre-trial settlement net of the legal costs can still exceed the pre-litigation settlement amount—leading to partial separation in the equilibria.

The existence of such equilibria with pre-litigation settlements relies on several key ingredients—(1) the defendant's perception of a sizable chance of involvement in high damage cases; (2) non-trivial legal costs for the defendant to wait until discovery in litigation; (3) the low cost of litigation for the plaintiff relative to the potential damages, such that the threat of litigation remains credible. Among other results, we show that plaintiffs with a higher potential legal damage can reach pre-litigation settlements while plaintiffs with lower damages cannot. They earn higher pre-litigation settlements, and receive higher settlements pre-trial—all key features that fit the distinction between portfolio PAEs and litigation PAEs. On the policy side, our results suggest that decreasing the defendant's cost to respond to litigation would increase the number of filed suits, but reduce cases of pre-litigation settlements. Moreover, if the early-stage litigation cost through discovery is lower than the later-stage costs of going to trial, then increasing the plaintiff's early-stage cost to file litigation reduces the number of cases filed for litigation, but drives more cases toward pre-litigation settlement; surprisingly, increasing the later-stage court cost will have the opposite effect by rendering litigation threats less credible.

Resolving disputes through pre-litigation settlements saves substantial legal costs, although it would create a mismatch between the settlement compensation and the plaintiff's real damages—a feature of pooling equilibrium. One the one hand, it allows the plaintiffs to receive compensation from low-damage cases that would have been too expensive to litigate. On the other hand, it leads some defendants with little fault to overpay for their true responsibility. The welfare trade-off is not obvious without imposing some preference for legal costs versus inefficiencies from mismatched transfers, a subject quite beyond the scope of the current article. However, the model advocates for caution toward policies exclusively aiming either at reducing pre-litigation settlement or at reducing the number of filed cases.

The rest of the article proceeds as follows. Section 2 summarizes the most relevant literature. Section 3 sets up of the model and analyzes it under the uniform distribution assumption to highlight the key intuitions of our results. Section 4 reports results under general distribution assumptions. Section 5 extends the baseline model to discuss the impact of adopting a fee-shifting rule. Section 6 discusses other extensions of the model and robustness, and Section 7 concludes.

## 2 Related Literature

A key feature of the pre-litigation settlement phase is the credibility of the threat to sue. Any entity can in principle send out demands at little cost; thus, at face value, their credibility in terms of threats of further legal actions is sub-par. Consequently, these cases introduce a credibility issue—why would the defendant pay a demand if the plaintiff may drop the case him or herself? Nalebuff (1987); Bebchuk (1988) and Meurer (1989) explore this topic in the presence of asymmetric information—a setting most closely related to ours. Nalebuff (1987) and Bebchuk (1988) study screening models where the uninformed party makes an offer to an informed party. Meurer (1989) analyzes a signaling setting where the rights to file a law suit reside with the uninformed defendant. The plaintiff in our setting has both the information advantage and the ability to threaten the defendant with a law suit. Consequently, our model produces different empirical implications.[2]

Another key feature of the pre-litigation settlement phase is free entry to sending a demand—anyone with a patent can send boiler plate demands with little additional costs. Reinganum and Wilde (1986) studies a model of pre-trial settlement in which an informed plaintiff sends a demand to an uninformed defendant. They found that, when plaintiffs of all types have credible cases, there exists a mixed strategy separating equilibrium in which the defendant plays a mixed strategy that rejects higher demands at a higher probability, thus inducing all plaintiffs to report their types truthfully. Farmer and Pecorino (2007) generalizes the model to include NEV cases, and shows that the plaintiffs with NEV cases have strong incentives to mimic offers from other types, therefore breaks down the proposed mixed strategy equilibrium when there is no entry cost of sending the demand. They also show that, if plaintiffs of all types have to pay a fixed fee before sending the demand, a mixed strategy separating equilibrium can be restored because, in the equilibrium, all plaintiffs with NEV cases will not enter. To our best knowledge, there is little known about whether litigants can reach settlement in the equilibrium and what cases move on to litigation when the plaintiff can send demands for free, i.e. some demands always come from plaintiffs with NEV cases—a characterizing feature of pre-litigation settlement. Our model fills this void by showing that partial pooling is still possible when there is free entry to the settlement bargaining game.

One strand of the legal studies literature focuses on the role of commitment and cost allocation—how strategically arranging the timing of legal costs may either enhance or deter the threat of "frivolous" law suits. On the one hand, the plaintiff can profitably extract a settlement by strategically front-loading legal costs (Bebchuk, 1996; Farmer and Pecorino, 1999; Chen, 2006). On the other hand, the defendant can also deter frivolous law suits by sinking cost through different legal contracts (Hubbard, 2016) or by establishing an aggressive reputation for rejecting settlements (Miceli, 1993; Bone, 1997). In the context of pre-litigation settlement, however, reputation or commitment alone is neither sufficient nor necessary to support or deter PAE activities—our theory proves the non-necessity and the behavior of many PAEs demonstrates insufficiency. Ewing and Feldman (2012) and Federal Trade Commission (2016) report a distinctive behavior of litigating PAEs, who intentionally group their patents into smaller pools and hide behind different shell companies before filing law suits. If playing tough or sinking legal costs was sufficient to extract settlements, we should not observe such behaviors.

Recent literature focuses specifically on the rise of the PAE phenomenon. Among theoretical analysis, Hovenkamp (2015) studies a screening model where a short-lived defendant may be a behavioral type that responds to aggressive plaintiffs by paying settlements. Thus, the PAE aggressively litigates to build a tough reputation to extract rents from future defendants. Choi and Gerlach (2015, 2017) study models where the

[2]We defer the detailed comparison of the predictions to Section 6.

existence of multiple defendants improves the expected return from individually non-credible law suits, thus providing incentives for the plaintiff to pursue litigation. The current model, however, sheds light on this topic from the perspective of pre-litigation settlement. In a sharp contrast, these models predict that PAEs with weaker patents file fewer law suits, while our model predicts the opposite—a result driven by the credibility of the threat to sue.

One key ingredient of our model is that the informed party may send a costless and unverifiable message to the uninformed party before incurring a cost to verify his or her own type, thus avoiding costs for both parties. Beyond law and economics, this feature renders our model a hybrid of a cheap talk game (Crawford and Sobel, 1982), where verification is impossible, and a lemon model with costly up-front voluntary disclosure (Verrecchia, 1983; Suijs, 1999), where costless communication before disclosure is impossible.

# 3 Model

## 3.1 Setup

A plaintiff (P or he) and an alleged defendant (D or she), both risk neutral, are involved in a dispute with an expected court judgment of $x$, which is randomly distributed on $[0, \overline{x}]$, where $\overline{x}$ may be infinity. $x$ has a cumulative distribution function $F(x)$ and probability distribution function $f(x)$, which we assume to be continuous and differentiable everywhere. Before deciding to file a law suit, P privately observes the true value of $x$, to which we sometimes refer as P's type. All of the above information is common knowledge to D, except for the realization of $x$.

Before filing suit, P may choose to send a costless message to D demanding a payment of $m = m(x)$ dollars, while $x$ cannot be credibly revealed to D. In other words, D or any third party cannot verify the validity of the demand at this stage. Without incurring any cost, D can choose to either respond by paying the demand (settle) or ignoring it (default). If D pays an amount acceptable to P, they reach a *pre-litigation* settlement agreement that releases D from further legal liabilities pertaining to this dispute. If P and D fail to reach an agreement, P may choose to incur a constant legal cost of $l_P$ to file suit, which consequently requires D to incur a constant legal cost of $l_D$ to respond. By spending $l_D$, D also acquires more information about $x$. For simplicity, we assume that the realization of $x$ is perfectly revealed to D after incurring $l_D$. Then, following a standard bargaining model under symmetric information, P and D may bargain and reach a *pre-court* settlement before incurring additional cost $k$ on each side to go to trial, where both parties expect the court to mandate that D pay P an amount equal to $x$.[3] There is no discounting between any of these stages in the model, and all legal costs are public knowledge.

To avoid the trivial scenario where none of the disputes has sufficient merit to pursue litigation, we assume that the upper bound of court value exceeds the higher value between litigation cost and trial cost, $\overline{x} > \max(l_P, k)$.[4]

**Subgame after D responds to the demand**

At trial, the expected payoffs are $-x - k - l_D$ for D and $x - k - l_P$ for P. Thus, before trial, P chooses to go to trial rather than dropping the case if and only if $x - k - l_p > -l_p$, i.e. $x > k$.

[3] Our results still hold when the legal costs are increasing in the judgment value $x$, see Appendix C for details.

[4] Under this assumption, it is still possible that all cases are "frivolous suits," i.e., they all have a negative expected value, $\overline{x} < l_P + k$.

The pre-trial settlement bargaining happens only if going to court is credible ($x > k$); thus, the payoffs after bargaining, assuming equal bargaining power, are

$$v_{PT}^{D} = \begin{cases} -x - l_D & \text{if } x > k \\ -l_D & \text{if } x \leq k \end{cases} \quad \text{and} \quad v_{PT}^{P} = \begin{cases} x - l_P & \text{if } x > k \\ -l_P & \text{if } x \leq k \end{cases},$$

where subscript $PT$ indicates pre-trial.

If D defaults on the demand, P decides to file a law suit if and only if $v_{PT}^{P} \geq 0$, which is equivalent to $x > \max(l_P, k)$. We refer to these as *law-suit-credible* cases, and others ($x \leq \max(l_P, k)$) as *non-credible* cases.[5] We define the shorthand $l_P^s = \max(l_P, k)$ to indicate the law suit-credibility threshold. In such a continuation game, P's pre-litigation demand strategies and D's responses constitute the core aspects of the equilibria with substantive demand. The rest of this section discusses potential equilibria in detail.

**Full separating equilibrium does not exist**

We first consider the possibility of a fully separating equilibrium, where Ps with different $x$ values send distinct pre-litigation demands $m(x)$, and show that this strategy does not constitute an equilibrium. If P adopts this strategy, D is updated about the true $x$ perfectly. A D who learns $x \leq l_P^s$ foresees that P will never credibly file the case, and thus refuses to pay the demand. This scenario yields a payoff of 0 for P with non-credible cases ($x \leq l_P^s$). On the other hand, a D who learns $x > l_P^s$ foresees the case being filed, and is thus willing to pay up to $x + l_D$ as a pre-litigation settlement. This latter scenario yields a payoff of $x + l_D$ for P with credible law suits ($x > l_P^s$), higher than the payoff from litigation, $x - l_P$. This outcome cannot be an equilibrium because Ps with non-credible cases have an incentive to deviate and mimic the strategies of Ps with credible cases to induce pre-litigation settlements from D. Indeed, Farmer and Pecorino (2007) shows that the same force from NEV cases also breaks down the mixed strategy separating equilibrium *à la* Reinganum and Wilde (1986). Because full separation is impossible, it does not loss generality to focus on pure strategy equilibria.

## 3.2 Equilibrium definition

**Definition.** A demand $m(x) > 0$ in the state of the world $x$ is *substantive* if, in equilibrium, it induces payment $m(x)$ from D, that is, the best response for any D who receives a demand is to pay the demanded amount rather than to default.

We adopt a standard pure-strategy perfect Bayesian Nash equilibrium (PBE) concept with one modification—all pre-litigation demands are substantive. We refer to this solution concept as *the equilibrium with substantive demand*. Note that there is always a no-demand equilibrium or "babbling equilibrium," where P never sends any substantive demand, and D never responds to pre-litigation demands.[6] However, we omit this type of equilibrium, and focus the analysis on the existence and properties of the equilibria with substantive demand.

**Definition.** A pure-strategy *PBE with substantive demand* is described by

- P's demand strategy $m(x) \in \varnothing \cup \mathbb{R}^+$ that maximizes his total payoff;

[5] We avoid using the term *frivolous law suit* here (Spier, 2007) because some cases may be law-suit credible, but still have a negative expected value if carried through to trial, i.e., $\max(l_P, k) < x \leq l_P + k$.

[6] One belief for D that would support such an equilibrium is that any demand indicates the lowest type of $x$.

- D's belief of the state of the world according to Bayes rule and to the correct belief of P's equilibrium demand strategy, $\tilde{f}(x) = f(x|m(x))$ whenever possible;
- D's binary response to demand $R(m(x)) \in \{\text{pay}, \text{default}\}$ that maximizes her total expected payoff;
- P's binary response to litigate $L(x, R) \in \{\text{litigate}, \text{not litigate}\}$ that maximizes his continuation payoff;
- P and D's payoff-maximizing settlement offers and responses given each partys strategies after the case is filed for suit;
- all pre-litigation demands are substantive.

We do not lose generality by focusing on equilibria with substantive demand. This additional requirement on the PBE frees the analysis from considering any additional haggling process leading up to the pre-litigation settlement or other fruitless communications before the suit is filed. Because D has no private information and $x$ is unverifiable, any signal besides $m(x)$ has no additional value. Suppose an equilibrium with demand $\tilde{m}(x)$ that induces D to pay $m(x)$ and settle with P pre-litigation; we can then construct an equilibrium with substantive demand to replicate the exact equilibrium outcome—P demands $m(x)$ directly and induces the exact pre-litigation settlement. Moreover, if, in equilibrium, P of a certain $x$ sends excessive demands that fail to induce payments from D, then we can also construct an equilibrium with substantive demand where such types of P do not send any demand to replicate the exact equilibrium path.[7]

## 3.3 Properties of equilibria

**Cutoff-pooling strategy in the equilibrium**

The discussion on separating equilibrium reveals that P with non-credible cases has strong incentives to mimic the strategies of those with law-suit-credible cases. Indeed, we show that any equilibrium with substantive demand must exhibit a feature of partial pooling, where all cases below a threshold reach the same pre-litigation settlement.

**Lemma 1.** *In any equilibrium with substantive demand, all Ps who send demands will send the same demand $m(x)$. Specifically, for $x_1 \neq x_2$, if $m(x_1) \neq \varnothing, m(x_2) \neq \varnothing$, then $m(x_1) = m(x_2)$.*

*Proof.* Suppose an equilibrium where P with $x_1$ and $x_2$ send two distinct messages $m(x_1) < m(x_2)$. If both demands are substantive, then the P of $x_1$ will deviate and send $m(x_2)$, a contradiction. □

Furthermore, if a P of particular type $\tilde{x}$ is willing to accept a given pre-litigation settlement, then all Ps with lower value cases must be willing to settle for the same amount.

**Lemma 2.** *In any equilibrium with substantive demand, if a P of type $\tilde{x}$ sends a substantive demand $m(\tilde{x})$, then a P with any $x < \tilde{x}$ must also send the same demand.*

*Proof.* Without reaching pre-litigation settlement, Ps of type $x < \tilde{x}$ receive less continuation payoff, if any, compared to Ps of type $\tilde{x}$. Specifically, between dropping the case before filing and moving on to reach pre-trial settlement, P's alternative payoff without pre-litigation settlement is $v^P_{PL}(x) = \max(0, v^P_{PT}(x)) = \max(0, x - l_P)$. Thus, $v^P_{PL}(x) \leq v^P_{PL}(\tilde{x})$ for all $x < \tilde{x}$, where subscript $PL$ represents pre-litigation.

[7] If we assume that P bears an infinitesimal cost to send a demand—it is presumably costly to make the demand look remotely credible, or a potential legal liability for sending demands in "bad faith," or risking having the patent invalidated by court—it would be in P's best interest to avoid such fruitless attempts and pursue litigation directly.

If a P of type $\tilde{x}$ sends substantive demand $m(\tilde{x})$ in the equilibrium, the demand $m(\tilde{x})$ must exceed the alternative payoff for a P of type $\tilde{x}$; thus, we have, for any $x < \tilde{x}$,

$$m(\tilde{x}) \geq v_{PL}^{P}(\tilde{x}) \geq v_{PL}^{P}(x),$$

which implies that all Ps of type $x$ have incentives to mimic type $\tilde{x}$ to send $m(\tilde{x})$ rather than bypassing pre-litigation settlement.

□

Following from Lemma 1 and 2, P must play a cutoff strategy in any equilibrium with substantive demand. This implies a simple structure to update beliefs on D's part.

If D received a pre-litigation demand, then holding the correct belief about P's strategy described by the cutoff $\tilde{x}$ in the equilibrium, D updates her belief about the case's trial judgment to the truncated distribution with p.d.f. $f(x|x \leq \tilde{x}) = \frac{f(x)}{F(\tilde{x})}$. Her expected continuation payoff in choosing to default is

$$\mathbb{E}\left[\mathbb{1}(x > l_P^s) v_{PT}^D | \tilde{x}\right] = \int_0^{\tilde{x}} \mathbb{1}(x > l_P^s) v_{PT}^D f(x|x \leq \tilde{x}) \mathrm{d}x = -\int_{l_P^s}^{\tilde{x}} (x + l_D) f(x|x \leq \tilde{x}) \mathrm{d}x, \quad (1)$$

where $\mathbb{1}(\cdot)$ is an indicator function. The expression always takes a negative value given our assumption that $\overline{x} > l_P^s$, indicating a real risk that the case may be filed for law suit if she refuses to pay the demand.[8] The upper bound of the resulting integral reflects the cutoff structure that only Ps with case values below $\tilde{x}$ would send a demand. Its lower bound $l_P^s$ reflects D's reasoning that only law-suit-credible cases will be filed for suit, leading to a continuation payoff of $x + l_D$ pre-trial, whereas all non-credible cases will be automatically dropped. This expression captures D's calculation that weighs the potential legal liability against the probability that the case will be filed for suit.

Given D's expected payoff from default, she is willing to pay up to $m(\tilde{x}) = -\mathbb{E}\left[\mathbb{1}(x > l_P^s) v_{PS}^D | \tilde{x}\right]$ to P as a pre-litigation settlement. If all Ps with $x \leq \tilde{x}$ are willing to accept this payment rather than filing the suit, i.e., $m(\tilde{x}) \geq x - l_P$, then $\tilde{x}$ summarizes an equilibrium with pre-litigation settlement, where all cases with a value below $\tilde{x}$ settle before being filed for suit for $m(\tilde{x})$, and all higher value cases proceed to law suit.

To complete the equilibrium description, we must impose requirements for off-equilibrium beliefs should D receive an excessive demand deviating from $m(\tilde{x})$. In such cases, as long as D updates her belief in such a way that lowers her willingness to pay for pre-litigation settlement, the equilibrium sustains. We summarize these key observations in the following proposition.

**Proposition 1.** *A cutoff demand strategy $\tilde{x} \in (l_P^s, \overline{x}]$ summarizes any equilibrium with substantive demand. In other words, without losing generality, we can focus on a cutoff strategy by P and the corresponding responses from D in the following form*

- *P sends demand $m(\tilde{x}) = -\int_{l_P^s}^{\tilde{x}} (x + l_D) f(x|x \leq \tilde{x}) dx$ for all $0 \leq x \leq \tilde{x}$ and files suit for all $x > \tilde{x}$;*
- *D, who receives the demand $m(\tilde{x})$, updates her belief about $x$ to $\tilde{f}(x) = f(x|x \leq \tilde{x})$;*
- *D is willing to pay $m(\tilde{x})$ in exchange for release from possible legal responsibilities;*
- *P with $x \leq \tilde{x}$ accepts pre-litigation settlement payments $m(\tilde{x})$; P with $x > \tilde{x}$ files a law suit directly; and*

[8] P playing $\tilde{x} < l_P^s$ is weakly dominated because it induces zero settlement payments from D.

- *D who receives any demand other than $m(\tilde{x})$ updates her belief to $\tilde{f}(x)|_{m\neq m(\tilde{x})}$, such that $\int_{l_P^s}^{\tilde{x}}(x+l_D)f(x|x\leq\tilde{x})dx \geq \int_{l_P^s}^{\tilde{x}}(x+l_D)\tilde{f}(x)|_{m\neq m(\tilde{x})}dx$.*

*Proof.* Implied by the previous discussion. □

For expositional purposes, we refer to any equilibrium with substantive demand by its cutoff strategy $\tilde{x}$, while implicitly assuming that Proposition 1 describes all other aspects of the equilibrium. To highlight the intuition of our key results, we proceed through the rest of this section with an example of a uniformly distributed $x$ on $[0,\overline{x}]$. With the uniform assumption, we can reach closed-form solutions to the cutoff strategy, and the example exhibits almost all of our key results. We defer the discussion of the results under general distribution assumptions until the next section.

**Full pooling equilibrium when $x$ is uniformly distributed**

We first consider the possibility of a full pooling equilibrium, where any type of P sends the same demand $m(x)=m$ and reaches pre-litigation settlement with D. If all types of P send identical demands, then receiving the message is completely uninformative; thus, D does not update at all. This is a special case where the cutoff strategy equals the upper bound of the distribution, $\tilde{x}=\overline{x}$. If D refuses to pay, her continuation payoff is, according to (1),

$$\mathbb{E}\left[\mathbb{1}(x>l_P^s)v_{PS}^D|\overline{x}\right] = -\int_{l_P^s}^{\tilde{x}}(x+l_D)\frac{1}{\overline{x}}\mathrm{d}x = -\frac{\overline{x}-l_P^s}{\overline{x}}\left(\frac{\overline{x}+l_P^s}{2}+l_D\right),$$

where the first term measures the probability that the case will be filed for suit, and the second term measures the average pre-trial legal liability plus Ds legal costs to reach that stage. The absolute value of this expression, $\left|\mathbb{E}\left[\mathbb{1}(x>l_P^s)v_{PS}^D|\overline{x}\right]\right|$ measures D's *willingness to pay for pre-litigation settlement*. Note that this expression is increasing and concave in the upper cutoff of the integral, $\overline{x}$ in this case—an essential property that shapes the properties of the equilibrium.

Given the consideration above, P can extract an amount up to $m=\left|\mathbb{E}\left[\mathbb{1}(x>l_P^s)v_{PS}^D|\overline{x}\right]\right|$ from D as pre-litigation settlement. It is incentive compatible for P to forego the law suit if $m$ exceeds his continuation payoff from filing the suit, $x-l_P$. Thus, the full pooling equilibrium holds if Ps of the highest type, $\overline{x}$, are willing to accept the settlement $m$ pre-litigation. Specifically, this requires

$$\frac{\overline{x}-l_P^s}{\overline{x}}\left(\frac{\overline{x}+l_P^s}{2}+l_D\right)\geq\overline{x}-l_P, \tag{2}$$

which, after some algebra, is equivalent to a quadratic expression in $\overline{x}$ with roots

$$x_L=(l_D+l_P)-\sqrt{(l_D+l_P)^2-l_P^s\cdot(2l_D+l_P^s)}$$
$$x_H=(l_D+l_P)+\sqrt{(l_D+l_P)^2-l_P^s\cdot(2l_D+l_P^s)}.$$

That is, if $\overline{x}\in[x_L,x_H]$, D's willingness to pay is greater than the continuation payoff from filing law suit for Ps of type $\overline{x}$, $m(\overline{x})\geq\overline{x}-l_P$. Note that $x_L,x_H$ are real if the following condition on the legal costs holds

$$l_D\geq(l_P^s-l_P)+\sqrt{2(l_P^s-l_P)\cdot l_P^s}, \tag{condition $l_D$}$$

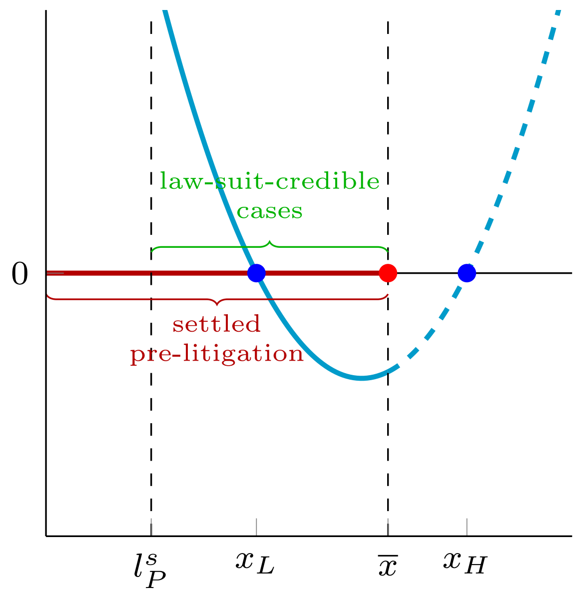



Figure 1: Full pre-litigation settlement ($\overline{x} \in [x_L, x_H]$)

which we can interpret as meaning that the defendant's pre-court litigation cost is relatively high compared to the plaintiff's pre-court litigation cost, and the cost to go to court (Figure 1).

**Proposition 2.** *Let $x$ be uniformly distributed on $[0, \overline{x}]$; then, there exists a full pooling equilibrium of substantive demand $\tilde{x} = \overline{x}$, where all cases reach pre-litigation settlement at the same demand, if*

- *condition $l_D$ holds; and*
- *the maximum court judgment lies within an intermediate range, i.e., $\overline{x} \in [x_L, x_H]$.*

*Proof.* Suppose D believes that Ps of all types send identical demands; then, D's willingness to pay for pre-litigation settlement is $m(\overline{x}) = \frac{\overline{x} - l_P^s}{\overline{x}} \left( \frac{\overline{x} + l_P^s}{2} + l_D \right)$. By the definition of $x_L, x_H$, we have

$$m(\overline{x}) \geq \overline{x} - l_P \geq x - l_P \quad \text{for all } x \in [0, \overline{x}],$$

which implies that all types of Ps have incentives to accept settlement $m(\overline{x})$ and forgo filling the suit. Therefore, P's best response, given D's belief and response strategy, is to demand $m(\overline{x})$ for all types. Hence, D's belief of a full pooling strategy is correct, and thus $\tilde{x} = \overline{x}$ is an equilibrium with substantive demand. □

Figure 1 illustrates the stated equilibrium in the proposition. The parabola in the figure is defined by (2). In this equilibrium, P of all types make an identical pre-litigation demand to D; although only a fraction of cases are law-suit-credible, D is willing to pay the demand. Therefore, all cases reach pre-litigation settlement and no case is filed for suit.

The first condition in the proposition is rather intuitive and echoes the arguments in the legal studies literature—high enough legal costs for the defendant relative to those of plaintiff lead to pre-litigation settlement. This insight generalizes beyond the uniform distribution assumption and the boundedness of $x$ (see Proposition 8). By reaching pre-litigation settlement, P gains on two fronts: avoiding additional legal cost $l_P$

while extracting $l_D$ from $D$. However, Ps with high case values also lose from foregoing litigation because the pre-litigation settlement is diluted by two forces—once by pooling with lower-value cases, and again by the credibility of the suit actually being filed. To make Ps incentive compatible to forego litigation, $l_D$ must sufficiently mitigate the gap between the diluted $m$ and his true $x$. Therefore, a relatively high $l_D$ is necessary to support the pooling outcome as an equilibrium.

The subtlety revealed by the first condition is that $l_P$ can play two different roles. On the one hand, an increase in $l_P$ can raise the law suit-credibility threshold $l_P^s$, rendering a given case less law-suit-credible for D, and lowering the settlement value $m$, thus making pre-litigation settlement less palatable for P. On the other hand, a higher $l_P$ makes pre-litigation settlement more enticing for P as it saves extra expenses by settling upfront. When $l_P$ is smaller than $k$, an increase in $l_P$ only has the second effect, which leads to a weaker condition to support the equilibrium above. However, when $l_P$ is greater than $k$, an increase in $l_P$ carries both effects, which counteract each other and can lead to ambiguous results. In the uniform example, these two forces exactly offset each other; thus, an increase in $l_P$ has no impact on the necessary condition for the equilibrium to hold.[9] Note, however, that these changes in legal costs do not change the pooling equilibrium outcome, $\tilde{x} = \overline{x}$, defined by the upper bound of the distribution.[10]

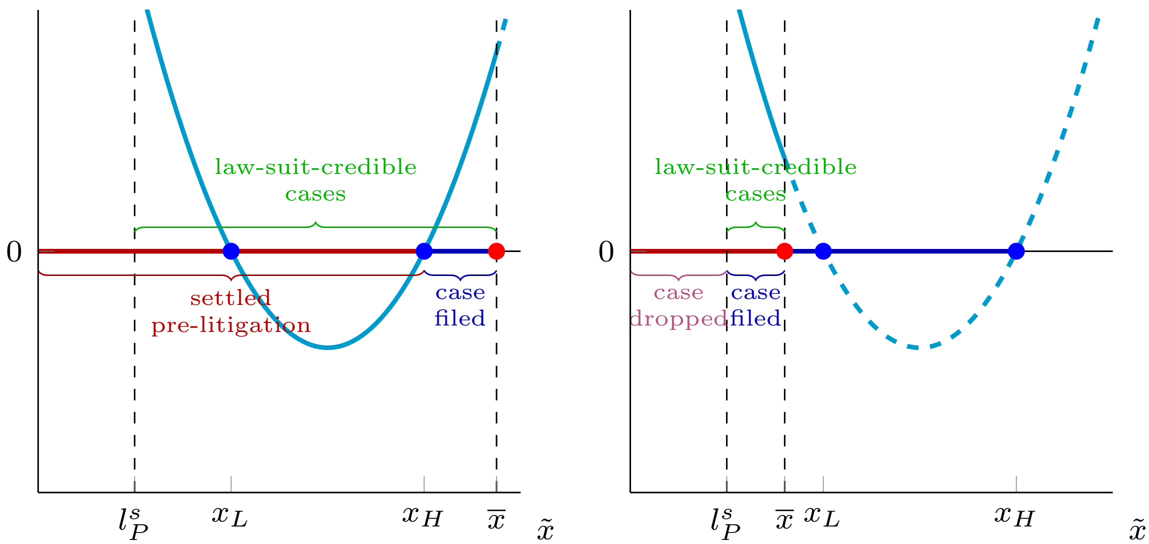
A closer look at the second condition in Proposition 2 raises two questions. First, why is there a possible interval $(l_P^s, x_L)$ in which the pooling equilibrium fails to hold if $\overline{x}$ falls into it? The intuition is that when $\overline{x}$ gets too close to $l_P^s$, the likelihood of a law suit becomes too low to support a settlement value $m$ attractive enough for any P with law-suit credible cases to forego the law suit, which consequently breaks the pooling equilibrium. In fact, if $\overline{x} < x_L$, there is no equilibrium with substantive demand—all cases above $l_P^s$ are filed for suit, and others are automatically dropped without settlement (see Proposition 3). The counter-intuitive observation in this case is that, affected by the low credibility of filing the suit, P will pursue litigation, although the value of the settlement is in fact relatively low—a result consistent with the behavior of litigation PAEs.

The second question is what happens if $\overline{x}$ exceeds $x_H$? Recall the key property that $m$ is increasing and concave in $\overline{x}$. When $\overline{x}$ exceeds $x_H$, the highest possible pre-litigation settlement $m$ from the full pooling outcome can no longer keep up with linearly-increasing case values at the higher end beyond $x_H$. This fact incentivizes P with such high value cases to deviate—bypassing the pre-litigation stage and going directly to a law suit. In other words, Ps with high-value cases would prefer going to litigation and waiting for D to discover $x$, thereby extracting a higher settlement pre-trial. In such cases, incurring litigation costs $l_P$ is worthwhile, thus bypassing demand stage is optimal. An ensuing question is whether there is still an equilibrium with substantive demand if $\overline{x}$ is greater than $x_H$. The answer is yes, and it takes the form of a partial pooling equilibrium indicated by Proposition 1. The rest of this section characterizes these equilibria and discusses their properties.

**Partial pooling equilibria when $x$ is uniformly distributed**

One important observation from the analysis above is that as the cutoff strategy $\tilde{x}$ ($\overline{x}$ in the full pooling scenario) decreases, D's willingness to pay may decrease slower than $\tilde{x}$ due to the concavity of $m(\tilde{x})$. Thus, even when the full pooling equilibrium is unsustainable, there may still exist a partial pooling equilibrium with a cutoff strategy $\tilde{x} < \overline{x}$.

[9] Specifically, if $l_P > k$, the first condition in Proposition 2 reduces to $l_D \geq 0$, where changes in $l_P$ have no effect.
[10] That is, unless $\overline{x}$ falls out of the interval $[x_L, x_H]$.




(a) Partial pre-litigation settlement ($\overline{x} > x_H$) (b) No pre-litigation settlement ($\overline{x} < x_L$)

Figure 2: Equilibria under uniform distribution

**Proposition 3.** *Let $x$ be uniformly distributed on $[0, \overline{x}]$ and suppose that condition-$l_D$ holds; then, the equilibria with substantive demand are characterized by the following cases*

- *if $\overline{x} > x_H$, $\tilde{x} = x_H$ is an equilibrium (partial pooling);*
- *if $x_L \le \overline{x} \le x_H$, $\overline{x}$ is an equilibrium (full pooling);*
- *if $\overline{x} < x_L$, no equilibrium with substantive demand exists.*

*Proof.* First, suppose $\overline{x} > x_H$; we then check that $\tilde{x} = x_H$ is an equilibrium. By the definition of $x_H$, $m(x_H) = x_H - l_P \ge x - l_P$ for all $x \le x_H$. Suppose D believes that $\tilde{x} = x_H$; then, D is willing to pay $m(x_H)$ as pre-litigation settlement. Given $m(x_H)$, Ps of type $x \le x_H$ are willing to settle. Moreover, $m(x_H) < x - l_P$ for all $x > x_H$; thus, Ps of type $x > x_H$ are unwilling to settle with payment $m(x_H)$. Therefore, $\tilde{x} = x_H$ is Ps best response, hence D's belief that $\tilde{x} = x_H$ is correct, and therefore $\tilde{x} = x_H$ is an equilibrium with substantive demand.

The second point reiterates proposition 2.

The third point follows from the fact that if $\overline{x} < x_L$, by (2), there is no $\tilde{x} \in [l_P^s, \overline{x}]$ such that $m(\tilde{x}) \ge \tilde{x} - l_P$. This implies that the most D is willing to pay for pre-litigation settlement is insufficient to encourage Ps with case values above $l_P^s$ to give up litigation, regardless of what $\tilde{x}$ P plays.

□

Figure 2 illustrates the two scenarios beyond the full pooling equilibrium. If $\overline{x} > x_H$, a partial pooling equilibrium exists in which all cases with $x \le x_H$ reach pre-litigation settlements (including cases that are not law suit-credible), and all cases above $x_H$ are filed for suit. If $\overline{x} < x_L$, no case can reach pre-litigation settlement, and only law suit-credible cases are filed for suit.

It is important to note that these are not all equilibria with substantive demand when $x$ is uniformly distributed. A similar argument as the proof above verifies that when $\overline{x} \ge x_L$, $\tilde{x} = x_L$ is also an equilibrium with substantive demand (more generally, see Proposition 4). Therefore, the equilibrium stated above may not be unique in each scenario.

However, we will show that, regardless of the distribution assumptions for $x$, the equilibrium with substantive demand that satisfies the intuitive criterion (Cho and Kreps, 1987) is always unique (Proposition 6). This equilibrium also maximizes the proportion of cases reaching pre-litigation settlement and P's net payoff among all equilibria. In the uniform case, they are the equilibria we stated in Proposition 3. Compare the two equilibria with substantive demand $x_L$ against $x_H$ in the case of $\overline{x} > x_H$. Suppose the equilibrium is $\tilde{x} = x_L$, and D only updates her beliefs accordingly and is willing to pay up to $m(x_L)$. If D suddenly receives a higher demand $m(x_H)$—a deviation from her expected equilibrium strategy—should she naively and stubbornly deem this an excessive demand and refuses to pay, or should she deem this a reasonable demand based on a credible higher-value case? The intuitive criterion imposes the additional requirement on the off-equilibrium belief such that upon receiving such a deviation, D is sophisticated—she is forced to conclude that $m(x_H)$ is in fact a credible threat of a higher-value case that lies between $[0, x_H]$, and is thus willing to pay such a higher settlement. In other words, equilibria such as $\tilde{x} = x_L$ are only sustainable due to the implicit assumption that D can credibly commit to only paying lower demands, even when a higher demand is credible. Between the assumption of strong commitment power for D versus the assumption that D is highly sophisticated in choosing her strategy to respond to the demand, we choose the latter. Therefore, we favor the equilibrium that satisfies the intuitive criterion. For its uniqueness, we call it *the equilibrium*, on which we focus our comparative statics analysis.

The last note on the equilibrium characterization under the uniform case is that, if exist, $x_L$ and $x_H$, or $x_L$ and $\overline{x}$ if $\overline{x} < x_H$, are all the equilibria with substantive demand. In fact, there exist at most two equilibria with substantive demand if $f(x)$ is log-concave, which includes uniform, normal, truncated normal, exponential, logistic, Weibull, Chi-squared, etc. (Proposition 5).

**Comparative Statics when $x$ is uniformly distributed**

The comparative statics hinge on the properties of $x_L$ and $x_H$ as the legal costs $l_D, l_P$ and $k$ change. If $\tilde{x} = \overline{x}$, the equilibrium does not respond to changes in the legal costs, although changes in legal costs may affect its existence condition stated in Proposition 2. The more interesting case lies in the partial pooling equilibrium, and we focus on analyzing the properties of $x_H$ rather than $x_L$ because it satisfies the intuitive criterion.

It is readily verifiable that $x_H$ is increasing in $l_D$.[11] The intuition reiterates our discussion of the full pooling equilibrium regarding the effect of $l_D$ on P's incentives—a higher $l_D$ raises D's willingness to pay and therefore attracts higher-value cases to settle without filing suit. The effects of $l_P$ and $k$ depend on whether $l_P$ or $k$ is greater, which determines the law suit-credibility threshold $l_P^s$.

If $l_P < k$, an increase in $l_P$ lowers the right-hand-side of (2) without changing the left-hand-side, making the condition easier to satisfy. Intuitively, it makes a given pre-litigation settlement more compelling for P because it avoids the higher cost of filing the suit. Therefore, in this case, an increase in $l_P$ always increases the equilibrium cutoff strategy $x_H$. On the other hand, an increase in $k$ does exactly the opposite—it lowers the left-hand-side of (2) without affecting its right-hand-side. That is, an increase in $k$ only increases the law suit-credibility threshold, $l_P^s$, which renders any given case less law-suit-credible, and consequently lowers D's willingness to pay for settlement. Hence, in the equilibrium, it drives more high-value cases toward law suit, if such an equilibrium still exists. These effects of legal costs on the equilibrium cutoff strategy also generalizes

[11] An increase in $l_D$ increases both the center and the width of the interval $[x_L, x_H]$. If $l_P < k$, the width is $2l_D$; otherwise, it is $\sqrt{(l_D + l_P)^2 - k(2l_D + k)}$, which increases in $l_D$ given condition-$l_D$.

to the amount of pre-litigation settlement $m(\tilde{x}^*)$ and the average pre-trial settlements more easily observed from data—they are weakly increasing in $l_P$ and $l_D$, and decreasing in $k$. Moreover, these observations are fully generalizable to all distribution assumptions (see Proposition 9 and 10, and Table 1).

If $l_P > k$, however, an increase in $l_P$ now carries both effects—lowering both sides of (2). In the uniform case, its effect in pushing P to settle dominates its effect in lowering D's willingness to pay, and thus $x_H$ is increasing in $l_P$. However, this unambiguous result can only generalize to distributions that are non-decreasing in $x$ (see Propositions 11, 12, and 13, and Table 2). In this case, an increase in $k$ has no effect on the equilibrium—it does not change P's trade-off between settling pre-litigation versus filing suit, nor does it affect the law-suit credibility of any case. Although the comparative statics results differ depending on the relationship between $l_P$ and $k$, in reality, scenarios under $l_P < k$ are likely to be more relevant because trial costs usually far outweigh the legal costs to file the suit and to go through pre-trial discovery.

The interplay between $l_D$ and $l_P$ in the existence of the equilibrium remains largely true, as the condition-$l_D$ highlights. As $l_D$ becomes sufficiently high, all cases can reach pre-litigation settlement (see Proposition 7). On the contrary, if we fix $l_D$ and $k$, then, raising $l_P$ sufficiently high can eradicate all equilibria with pre-litigation settlements, while, surprisingly, also leaving nearly no cases filed for suit—a dreamland for D and devastation for P (Proposition 8).

In addition to the effects of changes in legal costs, we also analyze how changes in the distribution of $x$ affects the equilibrium behavior. We can imagine that if D expects that a higher legal liability is more likely, her willingness to pay for pre-litigation settlement would be higher and thus lead to a higher equilibrium cutoff strategy. In the uniform case, changing the distributions upper bound $\overline{x}$ only affects the equilibrium when it moves below $x_H$. Generalizing beyond the uniform distribution, if one distribution likelihood ratio dominates another, it has a weakly higher equilibrium cutoff strategy (see Propositions 14 and 15). This implies that Ps with potential for higher legal damages settle higher-value cases pre-litigation, and potentially bring fewer suits—a result consistent with both the intuition and the distinctive behaviors of litigation and portfolio PAEs.

# 4 General Results

In this section, we formally characterize the equilibria with substantive demand and analyze their properties without resorting to the uniform assumption for the probability distribution.

## 4.1 Equilibrium characterization

Proposition 1 implies that, without loss of generality, we can focus on a cutoff strategy, $\tilde{x}$, by P. Therefore, P's pre-litigation demand problem is to choose $\tilde{x}$ and a corresponding demand $m(x)$ to maximize his total payoff from pre-litigation settlement and pre-trial return.

$$\max_{m(x),\tilde{x}} \mathbb{1}[x \le \tilde{x}]m(x) + \mathbb{1}[x > \tilde{x}](x - l_P)$$
$$\text{s.t.} \quad m(x) \le \int_{l_P^s}^{\hat{\tilde{x}}} (x + l_D) f(x|x \le \hat{\tilde{x}}) \mathrm{d}x,$$

where $\hat{\tilde{x}}$ is D's belief about $\tilde{x}$, and according to Bayes rule, $f(x|x \le \hat{\tilde{x}}) = \frac{f(x)}{F(\hat{\tilde{x}})}$ is the truncated distribution of $f(x)$ on $[0, \hat{\tilde{x}}]$. This program is equivalent to

$$\begin{aligned} &\max_{m(x),\tilde{x}} m(x) \\ \text{s.t.} \quad & m(x) \le \int_{l_P^s}^{\hat{\tilde{x}}} (x + l_D) f(x|x \le \hat{\tilde{x}}) \mathrm{d}x && \text{IC for D} \\ & m(x) \ge x - l_P \quad \forall x \le \tilde{x} && \text{IC for P.} \end{aligned}$$

In equilibrium, D holds the correct beliefs about P's strategy, $\hat{\tilde{x}} = \tilde{x}$. Then, these constraints jointly imply the following necessary condition for the equilibrium cutoff strategy $\tilde{x}$

$$\int_{l_P^s}^{\tilde{x}} (x + l_D) f(x|x \le \tilde{x}) \mathrm{d}x \ge \tilde{x} - l_P.$$

Note that P's demand in equilibrium, $m(\tilde{x}) = \int_{l_P^s}^{\tilde{x}} (x + l_D) f(x|x \le \tilde{x}) \mathrm{d}x$, is monotone increasing in $\tilde{x}$.[12] By this necessary condition, the equilibria must belong to the following set

$$\tilde{X} \equiv \left\{ l_P^s \le \tilde{x} \le \overline{x} : \int_{l_P^s}^{\tilde{x}} (x + l_D) f(x|x \le \tilde{x}) \mathrm{d}x \ge \tilde{x} - l_P \right\}. \tag{3}$$

In particular, one of the candidate equilibria maximizes P's payoff

$$\tilde{x}^* \equiv \max \left\{ l_P^s \le \tilde{x} \le \overline{x} : \int_{l_P^s}^{\tilde{x}} (x + l_D) f(x|x \le \tilde{x}) \mathrm{d}x \ge \tilde{x} - l_P \right\}.$$

In several occasions during our analysis, it is more convenient to work with an alternative form of $\tilde{X}$, which we can write as follows.

**Lemma 3.** *The set $\tilde{X}$ can be rewritten as*

$$\tilde{X} = \left\{ l_P^s \le \tilde{x} \le \overline{x} : \underbrace{\frac{\int_0^{\tilde{x}} F(x) dx}{F(\tilde{x})} + \frac{C}{F(\tilde{x})}}_{\equiv \Phi(\tilde{x})} \le l_P + l_D \right\}, \tag{4}$$

*where $C = \int_0^{l_P^s} x f(x) dx + l_D F(l_P^s)$ is constant with respect to $\tilde{x}$ and is non-negative.*

*Proof.* See appendix. □

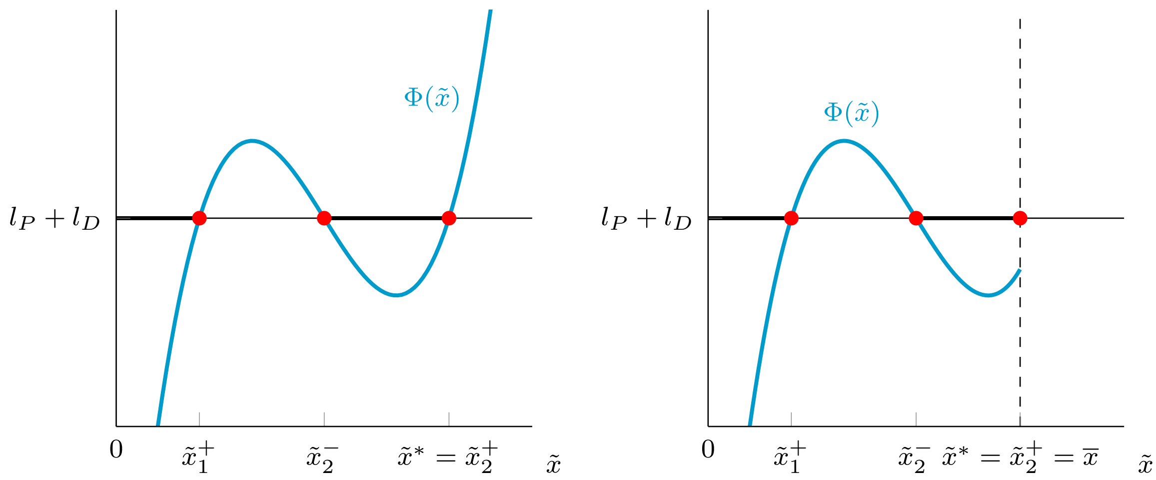
[12] To see this, integrating by parts, we have

$$\int_{l_P^s}^{\tilde{x}} (x + l_D) f(x|x \le \tilde{x}) \mathrm{d}x = \frac{(l_D + \tilde{x}) F(\tilde{x}) - (l_P^s + l_D) F(l_P^s) - \int_{l_P + \Delta}^{\tilde{x}} F(x)\, dx}{F(\tilde{x})},$$

of which we take derivative with respect to $\tilde{x}$, yielding

$$\frac{\partial}{\partial \tilde{x}} \left[ \int_{l_P^s}^{\tilde{x}} (x + l_D) f(x|x \le \tilde{x}) \mathrm{d}x \right] = \frac{f(\tilde{x}) \left( (l_D + l_P^s) F(l_P^s) + \int_{l_P^s}^{\tilde{x}} F(x)\, dx \right)}{F(\tilde{x})^2} \ge 0.$$

More generally, see Lemma 7.




(a) $\tilde{x}^* < \overline{x}$ (b) $\tilde{x}^* = \overline{x}$

Figure 3: Illustration of set $\tilde{X}$

By definition, we can write $\tilde{X}$ as a union of closed and convex subsets, $\tilde{X} = \cup_i X_i$. We denote the greatest and smallest elements of these subsets by $x_i^+$ and $x_i^-$, respectively. Figure 3 illustrates $\tilde{X}$ as defined by (4), as well as the associated $x_i^+$s and $x_i^-$s. Proposition 4 shows that all $x_i^+$s and $x_i^-$s constitute all equilibria with substantive demand in this model. In the case illustrated by Figure 3, suppose $l_P^s < \tilde{x}_1^+$; then, there exist three equilibria with substantive demand.

**Proposition 4.** *All equilibria with substantive demand are characterized by $x_i^+$ and $x_i^-$ in every convex subset of $\tilde{X}$, provided that $\tilde{X}$ is non-empty.*

*Proof.* See appendix. □

In addition, if $f(x)$ is log-concave, the set of potential equilibria is greatly simplified. Examples of such distributions include uniform, normal, truncated normal, exponential, logistic, Weibull, Chi-squared, and so on. (Bagnoli and Bergstrom, 2004).

**Lemma 4.** *The set $\tilde{X}$ is convex if $f(x)$ is log-concave.*

*Proof.* See appendix. □

**Proposition 5.** *There exist at most two equilibria with substantive demand if $f(x)$ is log-concave.*

*Proof.* By Lemma 4, $\tilde{X}$ is convex; thus, $x_i^+$ and $x_i^-$ are unique, if they exist. The result then follows from Proposition 4. □

**Proposition 6.** *If $\tilde{X}$ is non-empty, then its greatest element $\tilde{x}^*$ characterizes the unique equilibrium with substantive demand that satisfies the intuitive criterion. Moreover, among all equilibria with substantive demand, $\tilde{x}^*$ maximizes the number of cases reaching pre-litigation settlement, the settlement amount P receives, and P's payoff.*

*Proof.* In any other equilibrium $\tilde{x} < \tilde{x}^*$, consider the following out-of-equilibrium deviation of P, whose type lies in $[0, \tilde{x}^*]$ to send demand $m(\tilde{x}^*) > m(\tilde{x})$.

Such a deviation is equilibrium dominated for all types above $\tilde{x}^*$, but profitable for all Ps below $\tilde{x}^*$. Therefore, by the intuitive criterion, D must update her belief that $x$ lies in $[0, \tilde{x}^*]$, and thus pays $m(\tilde{x}^*)$, greater than the equilibrium demand $m(\tilde{x})$. The equilibrium thus fails to meet the intuitive criterion.

The same argument also shows that an equilibrium with cutoff strategy $\tilde{x}^*$ satisfies the intuitive criterion; thus the uniqueness. The fact that $\tilde{x}^*$ maximizes $m(\tilde{x})$ follows from the fact that $m(x)$ is increasing in $x$ (footnote 12). □

The following results highlight the properties of the equilibrium for extreme cases of the parameters.

**Proposition 7.** *Given $l_P^s$, when $l_D$ is sufficiently large, all cases reach pre-litigation settlement in the equilibrium.*

*Proof.* See appendix. □

**Proposition 8.** *If $\overline{x}$ is infinity, $f(x)$ is monotone decreasing for a large enough $x$; then, for fixed $l_D$ and sufficiently large $l_P$ or $k$, there exists no equilibrium with pre-litigation settlements, while the probability of a case being filed for litigation, $1 - F(l_P^s)$, goes to 0.*

*Proof.* See appendix. □

Propositions 7 and 8 provide a stark contrast against each other. When the cost of using the legal system is extremely high, parties involved in disputes are driven out of the formal litigation process—a common result following both the high cost for both P and D. However, the outcome under the shadow of high legal costs is completely different—when the legal cost for P or trial cost is exceptionally high, no pre-litigation settlement can be reached, and there is acquiesce. However, if the legal cost for D is exceptionally high, almost all cases are settled outside of the legal system.

## 4.2 Comparative statics

In the rest of the analysis, we focus on the unique equilibrium with substantive demand $\tilde{x}^*$ that satisfies the intuitive criterion, or *the equilibrium* for short. We analyze the effects of changes in the legal costs and the distribution of case values while assuming the existence of the equilibrium.

**Proposition 9.** *If P's litigation cost is less than the trial cost, $l_P < k$, then in the equilibrium, $\tilde{x}^*$ is non-decreasing if*

- *$l_D$ increases;*
- *$l_P$ increases; or*
- *$k$ decreases.*

*Proof.* See appendix. □

Figure 4 illustrates the idea behind our comparative statics analysis. Suppose a change in a parameter, say $l_P$, lowers the function $\Phi(\tilde{x}^*) - l_P - l_D$; it would then result in an expansion of the set $\tilde{X}$, and thus an increase in $\tilde{x}^*$ in a weak sense. Therefore, the response of $\Phi(\tilde{x}^*) - l_P - l_D$ with respect to changes in each parameter is sufficient in determining the responses of $\tilde{x}^*$.

Given the fixed distribution $F(x)$, a higher cutoff strategy in the equilibrium $\tilde{x}^*$ is also equivalent to a higher percentage of cases reaching pre-litigation settlement. Moreover, these results also extend to the pre-litigation settlement amount and the average pre-trial settlement amount in the equilibrium.

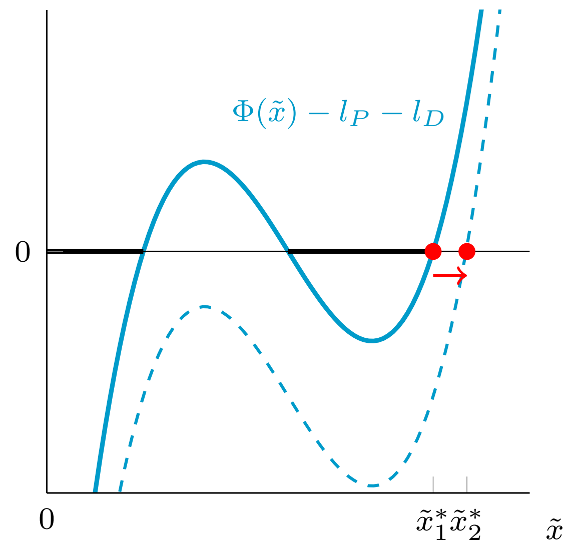



Figure 4: Comparative statics illustration

**Proposition 10.** *If P's litigation cost is less than the trial cost, $l_P < k$, then in the equilibrium, the pre-litigation settlement amount, $m(\tilde{x}^*)$, and the average pre-trial settlement amount, $\int_{\tilde{x}^*}^{\overline{x}} x f(x|x > \tilde{x}^*)dx$, are non-decreasing if*

- *$l_D$ increases;*
- *$l_P$ increases; or*
- *$k$ decreases.*

*Proof.* See appendix. □

We summarize these results in Table 1. The particularly surprising finding here, again, is the opposing effects of $l_P$ and $k$. The intuition, as we mentioned in the uniform example, lies in the fact that when $l_P < k$, an increase in $l_P$ attracts P with a given case to reach pre-litigation settlement. On the contrary, an increase in $k$ raises the law suit-credibility threshold $l_P^s$ and lowers D's willingness to pay for pre-litigation settlement, therefore driving more high-value cases toward law suit.

We summarize the comparative statics results under the less realistic assumption of $l_P > k$ below.

**Proposition 11.** *If P's litigation cost is greater than the trial cost, $l_P > k$, then in the equilibrium, $\tilde{x}^*$ is*

- *non-decreasing if $l_D$ increases;*
- *ambiguous if $l_P$ increases; and*
- *unresponsive to changes in $k$.*

*Proof.* Implied by the proof of Proposition 9. □

Similarly, these results extend to the settlement amounts, both pre-litigation and pre-trial.

**Proposition 12.** *If P's litigation cost is higher than the trial cost, $l_P > k$, then in the equilibrium, the pre-litigation settlement amount $m(\tilde{x}^*)$, and the observed average pre-trial settlement amount, $\int_{\tilde{x}^*}^{\overline{x}} x f(x|x > \tilde{x}^*)dx$,*

- *are non-decreasing if $l_D$ increases;*

Table 1: Policy Implications of Changes in Legal Cost (If $l_P < k$)

| Policy | # of cases filed for litigation | # of cases reach pre-litigation settlement | Pre-litigation settlement amount | Average pre-trial settlement amount |
|---|---|---|---|---|
| **decrease** Defendant's cost to respond to litigation, $l_D$ | ↑ | ↓ | ↓ | ↓ |
| **increase** Plaintiff's cost to file litigation, $l_P$ | ↓ | ↑ | ↑ | ↑ |
| **increase** Trial cost, $k$ | ↑ | ↓ | ↓ | ↓ |

- *are ambiguous if $l_P$ increases; and*
- *do not respond to changes in $k$.*

*Proof.* Omitted for its similarity to Proposition 10. □

To illustrate the second point, in which the prediction is ambiguous, we consider an example where $f(x)$ is exponentially distributed with a parameter of 1. In this case, we can verify that the effect of an increase in $l_P$ can go either way in the equilibrium.[13]

Similarly, we summarize the interpretations of the proposition in Table 2.

Table 2: Policy Implications of Changes in Legal Cost (If $l_P > k$)

| Policy | # of Cases file for litigation | # of Cases reach pre-litigation settlement | Pre-litigation settlement amount | Average pre-trial settlement amount |
|---|---|---|---|---|
| **decrease** Defendant's cost to respond to litigation, $l_D$ | ↑ | ↓ | ↓ | ↓ |
| **increase** Plaintiff's cost to file litigation, $l_P$ | ? | ? | ? | ? |
| **increase** Trial cost, $k$ | - | - | - | - |

We complement the previous statement with a technical result, where the effect of an increase in $l_P$ on $\tilde{x}^*$ is definitely non-decreasing. We do not think the premise of the result is necessarily realistic; however, we

[13]Specifically, let $f(x) = \exp(-x)$ and $F(x) = 1 - \exp(-x)$; then, for $l_P = 0.2, l_D = 1$, we have $\tilde{x}^* \approx 1.52$, and $\frac{\partial \tilde{x}^*}{\partial l_P} \approx -0.39 < 0$. Whereas if $l_P = 0.2, l_D = 0.2$, we have $\tilde{x}^* \approx 0.48$, and $\frac{\partial \tilde{x}^*}{\partial l_P} \approx 0.4 > 0$.

provide it below for the insights it offers into the technical properties of the equilibrium behavior, as well as its implication for the unambiguous comparative statics result in the uniformly distributed example—a special case that satisfies the stated conditions below.

**Proposition 13.** *Suppose $f(x)$ is non-decreasing in $x$. If P's litigation cost is higher than the trial cost, i.e., $l_P > k$, then $\tilde{x}^*$ is non-decreasing if $l_P$ increases.*

*Proof.* See appendix. □

Next, we study the effects of a change in the distribution of the case value $f(x)$. First, we state three technical lemmas that facilitate the proof of the following propositions.

**Definition.** A probability distribution function, $f(x)$, *likelihood ratio dominates* another, $g(x)$, if $\frac{f(x_2)}{f(x_1)} > \frac{g(x_2)}{g(x_1)}$ for all $x_2 > x_1$.

**Lemma 5.** *If $f(x)$ likelihood ratio dominates $g(x)$, then the same relation holds for their truncated distributions. In particular, $f(x|x \leq \tilde{x})$ likelihood ratio dominates $g(x|x \leq \tilde{x})$ and $f(x|x > \tilde{x})$ likelihood ratio dominates $g(x|x > \tilde{x})$.*

*Proof.* See appendix. □

**Lemma 6.** *If $f(x)$ likelihood ratio dominates $g(x)$, then for any $0 \leq l < \overline{x}$ and any positive and non-decreasing function $u(x)$,*

$$\int_l^{\overline{x}} u(x)f(x)dx \geq \int_l^{\overline{x}} u(x)g(x)dx.$$

*Proof.* See appendix. □

**Lemma 7.** *For any positive and non-decreasing function $u(x)$, probability distribution function $f(x)$ on $[0,\overline{x}]$, and $0 \leq l \leq t \leq \overline{x}$, we have*

- *$\int_l^t u(x)f(x|x \leq t)dx$ is non-decreasing in $t$; and*
- *$\int_l^{\overline{x}} u(x)f(x|x > l)dx$ is non-decreasing in $l$.*

*Proof.* See appendix. □

**Proposition 14.** *Let $f(x)$ and $g(x)$ be two probability density functions such that $f(x)$ likelihood ratio dominates $g(x)$; then,*

- *the equilibrium $\tilde{x}^*$ associated with $f(x)$ is weakly greater than that with $g(x)$;*
- *the pre-litigation settlement under $f(x)$ is weakly higher than that under $g(x)$;*
- *the average pre-trial settlement under $f(x)$ is weakly higher than that under $g(x)$.*

*Proof.* By Lemma 5, $f(x|x \leq \tilde{x})$ likelihood ratio dominates $g(x|x \leq \tilde{x})$.

Lemma 6 then applies to the truncated distributions $f(x|x \leq \tilde{x})$ and $g(x|x \leq \tilde{x})$. Let $l = l_P^s$ and $u(x) = x + l_D$; we then have

$$\int_{l_P^s}^{\tilde{x}} (x + l_D)f(x|x \leq \tilde{x})\mathrm{d}x \geq \int_{l_P^s}^{\tilde{x}} (x + l_D)g(x|x \leq \tilde{x})\mathrm{d}x,$$

which shows, given the same $\tilde{x}$, that D's willingness to pay the pre-litigation settlement under $f(x)$ is higher than that under $g(x)$.

Therefore, by (3), the $\tilde{X}$ associated with $g(x)$ is a subset of that with $f(x)$, and thus by Proposition 6, the equilibrium $\tilde{x}^*$ under $f(x)$, $\tilde{x}^*_f$, is weakly greater than that under $g(x)$, $\tilde{x}^*_g$.

Thus, by the first point in Lemma 7 and Lemma 6,

$$\int_{l_P^s}^{\tilde{x}^*_f}(x+l_D)f(x|x\le\tilde{x}^*_f)\mathrm{d}x \ge \int_{l_P^s}^{\tilde{x}^*_g}(x+l_D)f(x|x\le\tilde{x}^*_g)\mathrm{d}x \ge \int_{l_P^s}^{\tilde{x}^*_g}(x+l_D)g(x|x\le\tilde{x}^*_g)\mathrm{d}x.$$

Similarly, for pre-trial settlement, by the second point in Lemma 7, we have

$$\int_{\tilde{x}^*_f}^{\overline{x}} xf(x|x>\tilde{x}^*_f)\mathrm{d}x \ge \int_{\tilde{x}^*_g}^{\overline{x}} xf(x|x>\tilde{x}^*_g)\mathrm{d}x \ge \int_{\tilde{x}^*_g}^{\overline{x}} xg(x|x>\tilde{x}^*_g)\mathrm{d}x.$$

□

**Definition.** A plaintiff has *higher potential legal damage* if the probability distribution of a court judgment, $f(x)$, likelihood ratio dominates that of another plaintiff, $g(x)$.

**Proposition 15.** *Under the same legal costs, a P with higher potential legal damages files law suits with higher expected legal damages, and earns both higher pre-litigation settlements and higher average pre-trial settlements.*

*Proof.* Follows from proposition 14. □

**Lemma 8.** *For any distribution of $x$, $f_0(x)$, such that its associated set $\tilde{X}_0$ is singular, the associated equilibrium with substantive demand, $\tilde{x}^*_0$, is unique.*

*Proof.* Follows from the definition of $\tilde{X}$ and the singularity of $\tilde{X}_0$. □

**Proposition 16.** *Given fixed legal costs, the equilibrium with substantive demand under distribution $f(x)$*

- *does not exist, or equals $\tilde{x}^*_0$ if $f_0(x)$ likelihood-ratio dominates $f(x)$;*
- *exists, and $\tilde{x}^* \ge \tilde{x}^*_0$ if $f(x)$ likelihood-ratio dominates $f_0(x)$.*

*Proof.* Consider the first point: by the proof of Proposition 14, the set $\tilde{X}$ associated with $f(x)$ is a subset of $\tilde{X}_0$; the result thus follows.

The second point follows similarly. □

Proposition 15 offers an explanation for the distinctive behaviors observed from litigation and portfolio PAEs, as well as their coexistence. Proposition 16 depicts a more extreme version of the scenario. Under the same legal costs, the proposition shows that a P with higher potential legal damage, i.e., with $f(x)$ likelihood ratio dominates $f_0(x)$, files a fraction of law suits with high expected legal damage, earns high pre-litigation settlements, and high average pre-trial settlements; while on the other hand, Ps with lower potential legal damage, i.e., with $f(x)$ likelihood ratio dominated by $f_0(x)$, resort to exclusively filing law suits for pre-trial settlements, filing cases with lower values, and receiving much lower pre-trial settlements amount.

## 5 Extension: Effects of Fee Shifting Rules

Under the American rule, litigants bear their own legal costs, while under the fee shifting rule, or the English rule, the losing party bears the legal costs of both sides. A strand of literature studies the effects of these two cost allocation rules on settlements, such as Snyder and Hughes (1990); Hughes and Snyder (1995). In particular, the legal studies literature suggests that the English rule may help deter frivolous law suits (Polinsky and Rubinfeld, 1998; Rhode, 2004). Specifically on the topic of PAEs, some models suggest that applying a fee shifting rule may help reduce the extent of predatory litigation (Hovenkamp, 2015), while comparison studies of PAE activities between the US and Europe argue that the fee-shifting rule seems to explain why PAE activities are much less pronounced in Europe (Love et al., 2016; Thumm and Gabison, 2016). This section analyzes an extension of our model to study the effects of the fee shifting rule on the equilibrium. In particular, we show that the fee-shifting rule applied in the US context may help reduce cases filed for litigation, although it alone may not be able to fully explain the inactivity of PAEs in Europe relative to the US.

To compare the fee shifting rule against the American rule on equal footing, we alter the setup of the model slightly. Let $p$ be the probability that P is expected to win the case in court, and $W$ be the expected award the court will assign to P if he wins. Moreover, let $W$ follow a random distribution with probability distribution function $h(W)$. Then, the expected case value for P is $x = pW$ with p.d.f. function $f(x) = ph(W)$. We assume $p$ and $h(W)$ are common knowledge before the case is filed, while only P observes the realization of $W$ before filing. After incurring filing cost $l_D$, D learns about $W$.

Under the American rule, our model applies directly; the set of candidate equilibria (3) remains the same. For convenience of later comparisons, we rewrite it as

$$\tilde{X} \equiv \left\{ l_P^s \leq \tilde{x} \leq \overline{x} : \int_{l_P^s}^{\tilde{x}} x f(x|x \leq \tilde{x}) \mathrm{d}x + \frac{F(\tilde{x}) - F(l_P^s)}{F(\tilde{x})} l_D \geq \tilde{x} - l_P \right\}. \tag{5}$$

Under the fee-shifting rule, we need to re-specify the payoffs of the extensive form game to reflect the influences of the probability that P is expected to win the case. Let $\Sigma = 2k + l_D + l_P$ denote the total legal costs of both parties. At trial, the expected return is $x - (1-p)\Sigma$ for P and $-x - p\Sigma$ for D. Therefore, P has an incentive to go to trial instead of dropping the case if $x - (1-p)\Sigma \geq 0$, which implies $x \geq (1-p)\Sigma$—one of two law-suit credibility constraints for P.

After the case is filed, P and D can reach pre-trial settlement under symmetric information. Assuming equal bargaining power, the expected pre-trial settlement for P is

$$x^E = \frac{1}{2}\left[x - (1-p)\Sigma\right] + \frac{1}{2}\left[x + p\Sigma\right] = x + (p - \frac{1}{2})\Sigma,$$

where the superscript $E$ indicates the English rule. This expression is greater than that under the American rule if and only if $p \geq \frac{1}{2}$. If P files the case, the expected payoff, net of legal cost, is $x^E - l_P$. This implies the second law-suit credibility constraint—filing a law suit is viable without pre-litigation settlement if $x^E \geq l_P$, which is equivalent to $x \geq l_P - (p - \frac{1}{2})\Sigma$. The combined law-suit credibility threshold is

$$x \geq \max\left(l_P - (p - \frac{1}{2})\Sigma, (1-p)\Sigma\right) \equiv l_P^{s,E},$$

where

$$l_P^{s,E} = \begin{cases} l_P - (p - \frac{1}{2})\Sigma & \text{if} \quad l_P \leq k + \frac{l_D + l_P}{2} \\ (1-p)\Sigma & \text{if} \quad l_P \geq k + \frac{l_D + l_P}{2} \end{cases}.$$

If $p = \frac{1}{2}$, then the expected settlement is identical to that under the American rule, $x^E = x$, but the law-suit credibility threshold, $l_P^{s,E}$ is weakly lower than that under the American rule, $l_P^s$.[14]

D's willingness to pay for pre-litigation settlement is

$$\int_{l_P^{s,E}}^{\tilde{x}^E} \left(x^E + l_D\right) f(x|x \leq \tilde{x}^E)\mathrm{d}x = \int_{l_P^{s,E}}^{\tilde{x}} x f(x|x \leq \tilde{x}^E)\mathrm{d}x + \frac{F(\tilde{x}^E) - F(l_P^{s,E})}{F(\tilde{x}^E)}\left[(p - \frac{1}{2})\Sigma + l_D\right],$$

where it is sufficient to know $x = pW$, and $f(x) = f(pW) = h(\frac{x}{p})$ is the distribution of the expected court judgment derived from $W$'s p.d.f. $h(\cdot)$.

The set for candidate equilibria is

$$\tilde{X}^E \equiv \left\{ l_P^{s,E} \leq \tilde{x}^E \leq \overline{x} : \int_{l_P^{s,E}}^{\tilde{x}^E} x f(x|x \leq \tilde{x}^E)\mathrm{d}x + \frac{F(\tilde{x}^E) - F(l_P^{s,E})}{F(\tilde{x}^E)}\left[(p - \frac{1}{2})\Sigma + l_D\right] \geq x + (p - \frac{1}{2})\Sigma - l_P \right\},$$

which is equivalent to

$$\tilde{X}^E \equiv \left\{ l_P^{s,E} \leq \tilde{x}^E \leq \overline{x} : \int_{l_P^{s,E}}^{\tilde{x}^E} x f(x|x \leq \tilde{x}^E)\mathrm{d}x + \frac{F(\tilde{x}^E) - F(l_P^{s,E})}{F(\tilde{x}^E)} \underbrace{\left[ -\frac{F(l_P^{s,E})}{F(\tilde{x}^E) - F(l_P^{s,E})}(p - \frac{1}{2})\Sigma + l_D \right]}_{\equiv l_D^E} \geq x - l_P \right\}, \quad (6)$$

Where we call $l_D^E$ the *pseudo filing cost for D* under the fee-shifting rule.

Note that (6) is identical to (5), except for two differences: (1) between $l_D^E$ and $l_D$, hence the term pseudo filing cost for D for $l_D^E$; and (2) between $l_P^{s,E}$ and $l_P^s$. These two differences allow us to compare their corresponding equilibrium strategies.

**Proposition 17.** *Suppose $\tilde{X}^E$ and $\tilde{X}$ are non-empty. If the litigants believe that the chance of P winning in court is less than 1/2, then switching from the American rule to the English rule, holding legal costs and distribution of case values constant, will*

- *increase the cases settled pre-litigation;*
- *reduce the cases filed for litigation.*

*The results are indeterminate if the litigants believe that the chance of P winning in court is greater than 1/2.*

*Proof.* Because $l_P^{s,E} \leq l_P^s$, for any fixed $\tilde{x} = \tilde{x}^E$, we have

$$\int_{l_P^{s,E}}^{\tilde{x}^E} x f(x|x \leq \tilde{x}^E)\mathrm{d}x \geq \int_{l_P^s}^{\tilde{x}} x f(x|x \leq \tilde{x})\mathrm{d}x.$$

Moreover, if $p < 1/2$, then $l_D^E > l_D$. Therefore, the left-hand-side of the inequality in (6) is greater than that in (5), which implies that $\tilde{X} \subseteq \tilde{X}^E$, and thus $\tilde{x}^{*,E} \geq \tilde{x}^*$. This proves the two points in the proposition. However, a similar argument cannot hold in reverse when $p > 1/2$; thus, the indeterminate result. □

[14]Specifically, if $p = \frac{1}{2}$, we have $l_P^{s,E} = \frac{1}{2}\Sigma = k + \frac{l_D + l_P}{2} \leq l_P = l_P^s$ when $l_P \geq k + \frac{l_D + l_P}{2}$ and $l_P^{s,E} = l_P \leq k = l_P^s$ when $l_P \leq k < k + \frac{l_D + l_P}{2}$. However, when $k \leq l_P \leq k + \frac{l_D + l_P}{2}$, we have the same threshold, $l_P^{s,E} = l_P^s = l_P$.

It is worth noting that we cannot extend these results to compare pre-litigation settlement amounts or the average pre-trial settlement amount under the two systems—the results depend on the legal costs, the value of $p$, and the specific distribution.

Data collected by Love et al. (2016) shows that in Germany, P wins most cases proceeding to trial, while the opposite is true for England and Wales. If the litigants form their beliefs regarding $p$ based on these observations, Proposition 17 suggests that the fee-shifting rule helps explain the low PAE activities in England and Wales (about 85 cases in 2008); however, its effect on Germany is ambiguous (about 800 cases in 2008).

PAEs in the US, on the other hand, have a much lower rate of success at winning in court (Love et al., 2016). Therefore, switching to a fee-shifting rule would reduce the number of cases filed for suit by driving more cases toward pre-litigation settlement. Because the implications of the fee-shifting rule is mixed in the context of European courts, the fee-shifting rule alone may not be able to explain the differences in PAE court activities between the US and Europe.

One of the highlights of our analysis is that cases filed for suit by PAEs are only the tip of the iceberg. A substantial proportion of PAE activities happen under the shadow of the legal system in the form of pre-litigation settlement. Deterring case filings by PAEs is not and should not be the sole objective in the legal systems design.

## 6 Discussion

### Results and empirical facts on NPE activity

The predictions of our model are highly consistent with the facts reported in Federal Trade Commission (2016). Two of their findings stand out in comparison to our results:

- All portfolio PAEs in the FTC's study sent demands to initiate patent licensing negotiations with potential licensees, and portfolio PAEs typically executed patent licenses without suing the licensee (only 29% of portfolio PAE licenses related to settled litigation). By contrast, only ten of the 18 litigation PAEs or their affiliates sent demands, and 93% of litigation PAE licenses followed a patent infringement lawsuit against the licensee.
- Litigation PAEs accounted for 91% of the reported licenses, but only 20% of the reported revenue, or approximately $800 million, while portfolio PAE licenses, which accounted for 9% of the study total, constituted 80% of the reported revenue, or approximately $3.2 billion.

Proposition 15 speaks directly to these two observations. Portfolio PAEs hold much larger patent pools and more valuable patents, and can thus extract significant pre-litigation settlements (or licensing fees). On the other hand, litigation PAEs with less valuable patents have a lower litigation cutoff in the equilibrium, and consequently, lower pre-litigation settlement amounts, assuming that the equilibrium with pre-litigation settlement exists at all (Proposition 16). With sufficiently low patent values, such an equilibrium may not even exist and this plaintiff will have to resort to litigating every case to extract pre-trial settlements.

Given the good fit of our results to the facts, our empirical predictions summarized in Tables 1 and 2 on the effects of legal costs as well as Proposition 17 on the fee-shifting rule provide credible and relevant insights into the likely outcomes of policy changes aimed at deterring PAE activities. Hopefully, our results will inspire further empirical investigations.

## Comparison of empirical predictions

In the existing literature on legal settlements involving the credibility to sue, Bebchuk (1988); Nalebuff (1987) and Meurer (1989) analyze models most closely resembling ours. Our setup, however, leads to different predictions that provide opportunities to distinguish these models empirically.

Meurer (1989) also studies a signaling model where an informed P (patentee) makes a demand to settle with an uninformed D (competitor). The key difference is that, in their model, D has the right to file suit (litigate to invalidate the patent). With two types of Ps (holding invalid or valid patents), the equilibrium exhibits a mixed strategy and is unique under intuitive criterion. In it, high-type Ps (valid-patent holders) refuse to license and induce D to file suit; low-type Ps (holders of an invalid patent) mix licensing and thus induce D to mix filing suits and settling. In particular, it predicts that increasing the legal cost decreases the probability of settlements and has unknown effects on the probability of a case being filed. Our model has quite different predictions (Tables 1 and 2).

A simple extension of Bebchuk (1988) can be seen as the duo of the current model—an uninformed D makes an offer to the informed P with a continuous type space. We carefully extend this model in Appendix B to treat pre-litigation and pre-trial settlements separately. The highlight of the result is that only the greater of $l_P$ and $k$ affects the equilibrium cutoff of pre-litigation settlement and the pre-trial settlement. In other words, in that model, under the more relevant case of $l_P < k$, an increase in P's litigation costs, $l_P$, has no effect on the number of cases filed for suit, nor on the pre-trial settlement amount; whereas in our model, under the same case of $l_P < k$ (Table 1), an increase in P's litigation costs leads to more cases reaching pre-litigation settlement with higher settlement amounts, less cases filed for suit, and higher average pre-trial settlements.

Nalebuff (1987) studies another screening model where an uninformed P makes a demand to an informed D. The results also make different predictions—an increase in P's trial costs increases settlement demand and decreases the probability of settlement, while an increase in D's trial costs increases settlement demand and has the same probability of settlements.

## Extensions and robustness

In this subsection, we discuss some immediate extensions of our model and how relaxing some assumptions might affect our results. The first set of extensions concerns how legal costs are introduced in the model; the second set concerns how the continuation game is structured once the case is filed for suit.

There are good reasons to believe that legal costs may not be constant—cases with higher judgment values are likely to be more complex in nature and thus require higher cost to discover the facts and gather evidence.[15] Appendix C addresses this alternative modeling choice and shows that our key results remain unchanged.

Although we introduced $l_P$ and $l_D$ as filing costs in the model, we may expand their interpretations. In fact, they include all legal costs for P and D before D collects significant amount of information to make a settlement decision with P after the case is filed. These costs may also include non-monetary costs, such as the opportunity cost of valuable managerial resources diverted toward the law suit, negative publicity, and lost funding opportunities (Bessen and Meurer, 2013; Kiebzak et al., 2016).

We assumed symmetric trial costs for litigants, yet allowing asymmetry in these costs does not affect the

[15] Federal Trade Commission (2016) reports in their study that portfolio PAE cases also took much longer to settle than litigation PAE cases: 66% of litigation PAE cases that settled during the study period settled within a year, while only 26% of portfolio PAE cases that settled during the study period settled within a year.

structure and results of the model. Let $k_P$ and $k_L$ be the trial costs for P and D, respectively; then, $k_P$ will determine the law-suit credibility threshold along with $l_P$. The model will involve rewriting the expected pre-trial settlement value to reflect the differences in $k_P$ and $k_D$, but the core results remain largely intact.

Along the same lines, we assumed perfect learning for D after the case is filed, which consequently leads to perfect pre-trial settlement, though the choice of alternative continuation games need not change the core results of the model. If, instead, we adopt a more sophisticated pre-trial bargaining model, such as in Spier (1992), it only changes the expected pre-trial settlement value, the trade-off facing litigants during the pre-litigation stage should remain robust.

## 7 Conclusion

This article presents a model in which an informed plaintiff may send a costless demand to an uninformed defendant with a threat to sue. The analysis focuses on the range of cases reaching pre-litigation settlements versus those proceeding to litigation. The model predicted results that are highly consistent with the stylized facts reported by recent studies of PAE behavior, and generated empirical predictions of some proposed policy interventions.

The model may be extended in several meaningful ways in future work. A major assumption embedded in our equilibrium definition is the high degree of rationality among litigants. Relaxing this assumption by introducing litigants with bounded rationality may generate fresh insights. We leave this attempt to future efforts.

Because cases in this model have no social value beyond their legal judgment value, a formal welfare analysis of the optimal legal costs seems void. Thus, we omitted a formal treatment in this respect. However, as the analysis revealed, pre-litigation settlement resolves the dispute without incurring private and public legal resources, but it also suffers from the mismatch between the true damage and the settlement transfer. We thus argue that the design of legal procedure should also account for the inefficiencies in the pre-litigation settlements while minimizing societys legal costs.

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# A Proofs

*Proof of Proposition 4.* We prove the statement in two parts. First, we show that the greatest and smallest element of each convex subset of $\tilde{X}$ is an equilibrium. Second, we show that any interior point cannot be an equilibrium. With these results, the existence result follows from the fact that $\tilde{X}$ is closed and bounded.

Given D's belief that $\hat{\tilde{x}} = x_i^+$ or $x_i^-$, the highest $m(x)$ she is willing to pay is

$$m(\hat{\tilde{x}}) = \int_{l_P^s}^{\hat{\tilde{x}}} (x + l_D) f(x | x \leq \hat{\tilde{x}}) \mathrm{d}x.$$

Given this response, the highest type of P willing to send such a demand is $\tilde{x}$ that satisfies

$$m(\hat{\tilde{x}}) = \tilde{x} - l_P,$$

which holds in equality if $\hat{\tilde{x}} = \tilde{x} = x_i^+$ or $x_i^-$ by the definition of $\tilde{X}$ in (3). Therefore, we proved that any $\tilde{x} = x_i^+$ or $x_i^-$, and its corresponding demand and beliefs constitute an equilibrium.

Next, we conclude the proof by showing that any interior points in $X_i$ is not an equilibrium. For any interior point $\tilde{x} \in (x_i^-, x_i^+)$, and D's correct belief $\hat{\tilde{x}} = \tilde{x}$, it must be that

$$m(\hat{\tilde{x}}) > \tilde{x} - l_P,$$

which implies P of type $\tilde{x} + \epsilon$ for a sufficiently small $\epsilon > 0$ can profitably deviate and send demand $m(\hat{\tilde{x}})$, which then renders D's belief $\hat{\tilde{x}} < \tilde{x} + \epsilon$ incorrect.

□

*Proof of Lemma 3.*

$$\begin{aligned}
& \int_{l_P^s}^{\tilde{x}} (x + l_D) f(x | x \leq \tilde{x}) \mathrm{d}x \geq \tilde{x} - l_P \\
\Leftrightarrow \quad & \tilde{x} - \frac{1}{F(\tilde{x})} \int_0^{\tilde{x}} x f(x) \mathrm{d}x + \frac{1}{F(\tilde{x})} \int_0^{l_P^s} x f(x) \mathrm{d}x - \frac{1}{F(\tilde{x})} \int_{l_P^s}^{\tilde{x}} l_D f(x) \mathrm{d}x \leq l_P \\
\Leftrightarrow \quad & \tilde{x} - \frac{\tilde{x} F(\tilde{x}) - \int_0^{\tilde{x}} F(x) \mathrm{d}x}{F(\tilde{x})} + \frac{1}{F(\tilde{x})} \int_0^{l_P^s} x f(x) \mathrm{d}x - \frac{l_D}{F(\tilde{x})} [F(\tilde{x}) - F(l_P^s)] \leq l_P \\
\Leftrightarrow \quad & \frac{\int_0^{\tilde{x}} F(x) \mathrm{d}x}{F(\tilde{x})} + \frac{1}{F(\tilde{x})} \underbrace{\left[ \int_0^{l_P^s} x f(x) \mathrm{d}x + l_D F(l_P^s) \right]}_{\equiv C} \leq l_P + l_D
\end{aligned}$$

□

*Proof of Lemma 4.* We define a shorthand $\Phi(\tilde{x}) \equiv \frac{\int_0^{\tilde{x}} F(x) \mathrm{d}x}{F(\tilde{x})} + \frac{C}{F(\tilde{x})}$ for the left-hand-side of the defining inequality in (4).

The convexity result follows easily if the function $\Phi$ is monotone. We complete the proof by showing that, in the case that $\Phi$ is not monotone, it must be quasi-convex. To achieve this, we first show that any decreasing portion of $\Phi$ must be convex, and then, we prove that this implies the quasi-convexity of $\Phi$, thus the convexity of the set stated in proposition.

$\Phi$ is non-monotone if and only if, for some $\tilde{x}$,

$$\frac{\mathrm{d}\Phi}{\mathrm{d}\tilde{x}} = \frac{F(\tilde{x})^2 - f(\tilde{x})\int_0^{\tilde{x}} F(x)\mathrm{d}x - f(\tilde{x})C}{F(\tilde{x})^2} \leq 0 \quad \Leftrightarrow \quad F(\tilde{x})^2 \leq f(\tilde{x})\left(\int_0^{\tilde{x}} F(x)\mathrm{d}x + C\right). \tag{A.1}$$

The second order derivative of $\Phi$ is

$$\begin{aligned}
\frac{\mathrm{d}^2\Phi}{\mathrm{d}\tilde{x}^2} &= \frac{F(\tilde{x})^2\left[2F(\tilde{x})f(\tilde{x}) - f'(\tilde{x})\int_0^{\tilde{x}} F(\tilde{x})\mathrm{d}x - f(\tilde{x})F(\tilde{x}) - f'(\tilde{x})C\right]}{F(\tilde{x})^4} \\
&\qquad - \frac{2F(\tilde{x})f(\tilde{x})\left[F(\tilde{x})^2 - f(\tilde{x})\int_0^{\tilde{x}} F(\tilde{x})\mathrm{d}x - f(\tilde{x})C\right]}{F(\tilde{x})^4} \\
&= \frac{1}{F(\tilde{x})^3}\left[2f(\tilde{x})^2\left(\int_0^{\tilde{x}} F(x)\mathrm{d}x + C\right) - F(\tilde{x})f'(\tilde{x})\left(\int_0^{\tilde{x}} F(x)\mathrm{d}x + C\right) - F(\tilde{x})^2 f(\tilde{x})\right] \\
&= \frac{1}{F(\tilde{x})^3}\left[\underbrace{\left(f(\tilde{x})^2 - F(\tilde{x})f'(\tilde{x})\right)}_{\geq 0 \text{ by log-concavity of } f(x)}\left(\int_0^{\tilde{x}} F(x)\mathrm{d}x + C\right) + f(\tilde{x})\underbrace{\left[f(\tilde{x})\left(\int_0^{\tilde{x}} F(x)\mathrm{d}x + C\right) - F(\tilde{x})^2\right]}_{\geq 0 \text{ by (A.1)}}\right] \geq 0,
\end{aligned}$$

where the result in the first brace related to log-concavity follows from Remark 3 in Bagnoli and Bergstrom (2004).

Because $\Phi(\tilde{x})$ is differentiable, and its decreasing portion is convex, then $\Phi$ has to be quasi-convex. Suppose not, then there exists two adjacent local minimums $\tilde{x}_0 < \tilde{x}_2$, and a local maximum $\tilde{x}_1$ in between, such that $\tilde{x}_0 < \tilde{x}_1 < \tilde{x}_2$ and $\Phi(\tilde{x}_1) > \max(\Phi(\tilde{x}_0), \Phi(\tilde{x}_2)) \geq \Phi(\tilde{x}_2)$. However, because the decreasing portion of $\Phi$ is convex, and $\Phi'(\tilde{x}_1) = \Phi'(\tilde{x}_2) = 0$, it must be that, for any $y$ such that $\tilde{x}_1 < y < \tilde{x}_2$, $\Phi'(y) = 0$. This then implies $\Phi(\tilde{x}_1) = \Phi(\tilde{x}_2)$, a contradiction.

□

*Proof of Proposition 7.* Collecting $l_D$ and rewrite (4) as

$$\left\{l_P^S \leq \tilde{x} \leq \overline{x} : \int_0^{\tilde{x}} F(x)\mathrm{d}x + \int_0^{l_P^s} xf(x)\mathrm{d}x - l_P F(\tilde{x}) + l_D\left[F(l_P^s) - F(\tilde{x})\right] \leq 0\right\}$$

Define $\underline{l_D} = \frac{1}{F(\tilde{x}) - F(l_P^S)}\left[\int_0^{\tilde{x}} F(x)\mathrm{d}x + \int_0^{l_P^s} xf(x)\mathrm{d}x - l_P F(\tilde{x})\right]$, then the above defining inequality holds for all $\tilde{x}$ if $l_D \geq \underline{l_D}$. This implies $\tilde{x}^*$ goes to $\overline{x}$ as $l_D$ goes to infinity.

□

*Proof of Proposition 8.* Rewrite the left-hand side of the defining inequality in (4) as

$$\begin{aligned}
&\int_0^{\tilde{x}} F(x)\mathrm{d}x + \int_0^{l_P^s} x f(x)\mathrm{d}x - l_P F(\tilde{x}) + l_D \left[F(l_P^s) - F(\tilde{x})\right] \\
&= \int_0^{\tilde{x}} F(x)\mathrm{d}x + l_P^s F(l_P^s) - \int_0^{l_P^s} F(x)\mathrm{d}x - l_P F(\tilde{x}) + l_D \left[F(l_P^s) - F(\tilde{x})\right] \\
&= \int_{l_P^s}^{\tilde{x}} F(x)\mathrm{d}x + l_P^s F(l_P^s) - l_P F(\tilde{x}) + l_D \left[F(l_P^s) - F(\tilde{x})\right] \\
&\geq \int_{l_P^s}^{\tilde{x}} F(x)\mathrm{d}x + l_P \left[F(l_P^s) - F(\tilde{x})\right] + l_D \left[F(l_P^s) - F(\tilde{x})\right] \\
&= \int_{l_P^s}^{\tilde{x}} F(x)\mathrm{d}x - l_P \int_{l_P^s}^{\tilde{x}} f(x)\mathrm{d}x - l_D \int_{l_P^s}^{\tilde{x}} f(x)\mathrm{d}x \\
&\geq \int_{l_P^s}^{\tilde{x}} F(x)\mathrm{d}x - \int_{l_P^s}^{\tilde{x}} x f(x)\mathrm{d}x - l_D \int_{l_P^s}^{\tilde{x}} f(x)\mathrm{d}x \\
&= \int_{l_P^s}^{\tilde{x}} \left[F(x) - x f(x) - l_D f(x)\right]\mathrm{d}x \quad \geq 0 \quad \text{for large enough } l_P^s,
\end{aligned}$$

where the last inequality holds because $F(x) - x f(x) - l_D f(x) \geq 0$ if $f(x)$ is monotone decreasing for sufficiently large $x$. This is true because $F(x) \to 1$ and $l_D f(x) \to 0$ as $x \to \infty$; moreover, we have $x f(x) \to 0$ because, for any arbitrarily small $\epsilon$ and large enough $x$

$$\epsilon > \int_x^{\infty} f(u)\mathrm{d}u \geq \int_x^{2x} f(u)\mathrm{d}u \geq \int_x^{2x} f(x)\mathrm{d}u = x f(x).$$

Therefore, $\tilde{X}$ is empty given the stated conditions. Furthermore, given that $l_P$ or $k$ goes to infinity, so does $l_P^s$. Thus, the probability of a case being filed for litigation, $1 - F(l_P^s)$, goes to 0.

□

*Proof of Proposition 9.* Because $\tilde{x}^*$ is the greatest element of $\tilde{X}$, which we can re-write, according to (4), as $\tilde{X} = \left\{l_P^s \leq \tilde{x} \leq \overline{x} : \Phi - l_P - l_D \leq 0\right\}$, thus $\tilde{x}^*$ is non-decreasing if $\Phi - l_P - l_D$, a function of $\tilde{x}$, decreases, and vice versa.

Therefore, we study the comparative statics by analyzing how $\Phi - l_P - l_D$ is affected by changes in the cost parameters.

$$\begin{aligned}
\frac{\partial}{\partial l_P}(\Phi - l_P - l_D) &= \frac{\partial}{\partial l_P}\left(\frac{\int_0^{\tilde{x}} F(x)\mathrm{d}x}{F(\tilde{x})} + \frac{C}{F(\tilde{x})} - l_P - l_D\right) \\
&= \frac{\partial}{\partial l_P}\left(\frac{\int_0^{\tilde{x}} F(x)\mathrm{d}x}{F(\tilde{x})} + \frac{\int_0^{l_P^s} x f(x)\mathrm{d}x + l_D F(l_P^s)}{F(\tilde{x})} - l_P - l_D\right) \\
&= \frac{\left(l_P^S + l_D\right) f(l_P^S)}{F(\tilde{x})}\frac{\partial l_P^S}{\partial l_P} - 1 \\
&= \begin{cases} \frac{(l_P + l_D) f(l_P)}{F(\tilde{x})} - 1 & \text{if } l_P \geq k \\ -1 & \text{if } l_P < k \end{cases}.
\end{aligned}$$

Similar exercises show that

$$\frac{\partial}{\partial k}(\Phi - l_P - l_D) = \frac{\left(l_P^S + l_D\right) f(l_P^S)}{F(\tilde{x})} \frac{\partial l_P^S}{\partial k}$$
$$= \begin{cases} \frac{(k+l_D) f(k)}{F(\tilde{x})} \geq 0 & \text{if } k \geq l_P \\ 0 & \text{if } k < l_P \end{cases},$$

and that

$$\frac{\partial}{\partial l_D}(\Phi - l_P - l_D) = \frac{F(l_P^S)}{F(\tilde{x})} - 1 \leq 0.$$

The results therefore established the statement for $\tilde{x}^*$.

□

*Proof of Proposition 10.* For $m(\tilde{x}^*) = \int_{l_P^s}^{\tilde{x}^*} (x + l_D) f(x|x \leq \tilde{x})\mathrm{d}x$, we have

$$\frac{\mathrm{d}m(\tilde{x}^*)}{\mathrm{d}l_P} = \underbrace{\frac{\partial m(\tilde{x}^*)}{\partial \tilde{x}^*}}_{\geq 0 \text{ by footnote 12}} \overbrace{\frac{\partial \tilde{x}^*}{\partial l_P}}^{\geq 0 \text{ by Prop. 9}} + \underbrace{\frac{\partial m(\tilde{x}^*)}{\partial l_P}}_{=0 \text{ if } l_P<k=l_P^s} \geq 0.$$

Similarly, we have

$$\frac{\mathrm{d}m(\tilde{x}^*)}{\mathrm{d}k} = \underbrace{\frac{\partial m(\tilde{x}^*)}{\partial \tilde{x}^*}}_{\geq 0 \text{ by footnote 12}} \overbrace{\frac{\partial \tilde{x}^*}{\partial k}}^{\leq 0 \text{ by Prop. 9}} + \underbrace{\frac{\partial m(\tilde{x}^*)}{\partial k}}_{\leq 0 \text{ if } k=l_P^s} \leq 0 \quad \text{and} \quad \frac{\mathrm{d}m(\tilde{x}^*)}{\mathrm{d}l_D} = \underbrace{\frac{\partial m(\tilde{x}^*)}{\partial \tilde{x}^*}}_{\geq 0 \text{ by footnote 12}} \overbrace{\frac{\partial \tilde{x}^*}{\partial l_D}}^{\geq 0 \text{ by Prop. 9}} + \underbrace{\frac{\partial m(\tilde{x}^*)}{\partial l_D}}_{\geq 0} \geq 0.$$

As of the average pre-trial settlement amount, $\int_{\tilde{x}^*}^{\overline{x}} x f(x|x > \tilde{x}^*)\mathrm{d}x$, we will show that it is increasing in $\tilde{x}^*$, then the result follows from Proposition 9.

$$\frac{\partial}{\partial \tilde{x}^*} \int_{\tilde{x}^*}^{\overline{x}} x f(x|x > \tilde{x}^*)\mathrm{d}x = \frac{\partial}{\partial \tilde{x}^*}\left[\frac{1}{1 - F(\tilde{x}^*)} \int_{\tilde{x}^*}^{\overline{x}} x f(x)\mathrm{d}x\right]$$
$$= \frac{[1 - F(\tilde{x}^*)][-\tilde{x}^* f(\tilde{x}^*)] + f(\tilde{x}^*) \int_{\tilde{x}^*}^{\overline{x}} x f(x)\mathrm{d}x}{[1 - F(\tilde{x}^*)]^2}$$
$$= \frac{f(\tilde{x}^*)}{[1 - F(\tilde{x}^*)]^2}\left[-\tilde{x}^*[1 - F(\tilde{x}^*)] + \int_{\tilde{x}^*}^{\overline{x}} x f(x)\mathrm{d}x\right]$$
$$= \frac{f(\tilde{x}^*)}{[1 - F(\tilde{x}^*)]^2}\left[-\int_{\tilde{x}^*}^{\overline{x}} \tilde{x}^* f(x)\mathrm{d}x + \int_{\tilde{x}^*}^{\overline{x}} x f(x)\mathrm{d}x\right]$$
$$\geq \frac{f(\tilde{x}^*)}{[1 - F(\tilde{x}^*)]^2}\left[-\int_{\tilde{x}^*}^{\overline{x}} x f(x)\mathrm{d}x + \int_{\tilde{x}^*}^{\overline{x}} x f(x)\mathrm{d}x\right] = 0.$$

□

*Proof of Proposition 13.* Based on the proof of Proposition 9, it remains to sign $\frac{\partial}{\partial l_P}(\Phi - l_P - l_D) = \frac{(l_P+l_D)f(l_P)}{F(\tilde{x})} - 1$.

Suppose $\tilde{x}^*$ belongs to the interior of $\tilde{X}$, then $\tilde{x}^* = \overline{x}$, which is unaffected by the changes in legal costs, thus our results hold trivially. Otherwise, we have $\tilde{x}^* = \max(\tilde{X})$, in which case, by Proposition 4 and 6, it must be true that (1) $\Phi'(\tilde{x}^*) \geq 0$; and (2) $\Phi(\tilde{x}^*) = l_P + l_D$. The first point implies that

$$\frac{\mathrm{d}\Phi}{\mathrm{d}\tilde{x}^*} = \frac{F(\tilde{x}^*)^2 - f(\tilde{x}^*)\left(\int_0^{\tilde{x}^*} F(x)\mathrm{d}x + C\right)}{F(\tilde{x}^*)^2} \geq 0 \quad \Rightarrow \quad F(\tilde{x}^*)^2 - f(\tilde{x}^*)\left(\int_0^{\tilde{x}^*} F(x)\mathrm{d}x + C\right) \geq 0.$$

While the second point implies that

$$\Phi(\tilde{x}^*) = l_P + l_D \quad \Rightarrow \quad \int_0^{\tilde{x}^*} F(x)\mathrm{d}x + C = F(\tilde{x}^*)[l_P + l_D].$$

Combining both points, we have

$$F(\tilde{x}^*)\left(F(\tilde{x}^*) - f(\tilde{x}^*)[l_P + l_D]\right) \geq 0 \quad \Rightarrow \quad \frac{(l_P + l_D)f(\tilde{x}^*)}{F(\tilde{x}^*)} - 1 \leq 0.$$

If $f(x)$ is non-decreasing in $x$, then we have $\frac{(l_P+l_D)f(l_P)}{F(\tilde{x}^*)} - 1 \leq \frac{(l_P+l_D)f(\tilde{x}^*)}{F(\tilde{x}^*)} - 1 \leq 0$ because $\tilde{x}^* > l_P$ if it exists. □

*Proof of Lemma 5.* Suppose $f(x)$ likelihood ratio dominates $g(x)$, then for any $x_2 > x_1$, and a truncation to the interval $[a, b]$

$$\frac{f(x_2)}{f(x_1)} > \frac{g(x_2)}{g(x_1)} \quad \Leftrightarrow \quad \frac{\frac{f(x_2)}{F(b)-F(a)}}{\frac{f(x_1)}{F(b)-F(a)}} > \frac{\frac{g(x_2)}{G(b)-G(a)}}{\frac{g(x_1)}{G(b)-G(a)}} \quad \Leftrightarrow \quad \frac{f(x_2|x \in [a,b])}{f(x_1|x \in [a,b])} > \frac{g(x_2|x \in [a,b])}{g(x_1|x \in [a,b])}.$$

Let $[a, b] = [0, \tilde{x}]$, we have $f(x|x \leq \tilde{x})$ likelihood ratio dominates $g(x|x \leq \tilde{x})$. Similarly, let $(a, b] = [\tilde{x}, \overline{x}]$, we have $f(x|x > \tilde{x})$ likelihood ratio dominates $g(x|x > \tilde{x})$.

□

*Proof of Lemma 6.* Because $f(x)$ likelihood ratio dominates $g(x)$, it is also true that $f(x)$ first order stochastic dominates $g(x)$, and therefore

$$\int_0^{\overline{x}} u(x)f(x)\mathrm{d}x \geq \int_0^{\overline{x}} u(x)g(x)\mathrm{d}x$$

for any increasing and positive function $u(x)$.

Define $\Delta(l) \equiv \int_l^{\overline{x}} u(x)f(x)\mathrm{d}x - \int_l^{\overline{x}} u(x)g(x)\mathrm{d}x$ as a function of $l$. Note that, in particular, $\Delta(0) = \int_0^{\overline{x}} u(x)f(x)\mathrm{d}x - \int_0^{\overline{x}} u(x)g(x)\mathrm{d}x \geq 0$.

Moreover, because $f(x)$ likelihood ratio dominates $g(x)$, there exists $l^*$ such that $f(l) \geq g(l)$ for all $l \geq l^*$ and vice versa.

If $l \geq l^*$, then $\Delta(l) \geq 0$ by definition of $\Delta$ and $u$.

If $0 \leq l < l^*$, then $\Delta(l)$ is increasing in $l$ since $\frac{\mathrm{d}\Delta}{\mathrm{d}l} = u(l)[g(l) - f(l)] > 0$. Therefore, $\Delta(l) \geq \Delta(0) \geq 0$.

We therefore conclude that

$$\int_l^{\overline{x}} u(x)f(x)\mathrm{d}x \geq \int_l^{\overline{x}} u(x)g(x)\mathrm{d}x.$$

□

*Proof of Lemma 7.*

$$\begin{aligned}\frac{\partial}{\partial t}\left[\int_l^t u(x)f(x|x\leq t)\mathrm{d}x\right] &= \frac{f(t)}{F(t)^2}\left[u(t)F(t)-\int_l^t u(x)f(x)\mathrm{d}x\right] \\ &= \frac{f(t)}{F(t)^2}\left[\int_0^t u(t)f(x)-\int_l^t u(x)f(x)\mathrm{d}x\right] \\ &\geq \frac{f(t)}{F(t)^2}\left[\int_l^t u(t)f(x)-\int_l^t u(x)f(x)\mathrm{d}x\right]\geq 0.\end{aligned}$$

Similarly,

$$\begin{aligned}\frac{\partial}{\partial l}\left[\int_l^{\overline{x}} u(x)f(x|x>l)\mathrm{d}x\right] &= \frac{f(l)}{[1-F(l)]^2}\left[\int_l^{\overline{x}} u(x)f(x)\mathrm{d}x-u(l)[1-F(l)]\right] \\ &= \frac{f(l)}{[1-F(l)]^2}\left[\int_l^{\overline{x}} u(x)f(x)\mathrm{d}x-u(l)\int_l^{\overline{x}} f(x)\mathrm{d}x\right]\geq 0.\end{aligned}$$

□

# B Alternative version - if the defendant makes offer

In this section, we consider an alternative setup as in the article—letting D make offer to P. This change turns our original signaling model into a screening model similar to that of Bebchuk (1988). Like our main signaling model, this version here is built upon a subgame of pre-trial settlement, where P, although filed suit, might not necessarily follow through trial. One technical difference from Bebchuk (1988) is that we treat the probability of a case being "frivolous" as jointly determined by the legal costs and the distribution of the judgment value, rather than a explicit parameter.

The analysis reveals that the screening version also exhibits a partial pooling equilibrium with a cutoff. However, one critical difference in the comparative statics result in the screening model is that the trial cost, $k$, has no effect on the equilibrium cutoff strategy, or the amount of pre-litigation settlement (Proposition B.2). This result is in sharp contrast with the predictions of the signaling version, thus provides an empirical opportunity to distinguish these two models.

Suppose, upon learning about P's intention to file suit, D makes the offer $s$ to settle with P. Without knowledge of the exact value of $x$, D makes the same offer to any P. The equilibrium must involve pooling below a cutoff because if P with case value $\tilde{x}$ is willing to accept $s$ and forego litigation, all P with lower case values will also accept the same settlement.

D chooses $s$ to solve the following program.

$$\begin{aligned} \min_s \quad & \mathbb{E}\Big[\mathbb{1}[x \le \tilde{x}(s)]s + \mathbb{1}[x > \tilde{x}(s)](x + l_D)\Big] \\ \text{s.t.} \quad & s \ge x - l_P^s \quad \text{for all } x \le \tilde{x}(s) && \text{(IC for P)} \\ & \mathbb{E}\Big[\mathbb{1}[x \le \tilde{x}(s)]s + \mathbb{1}[x > \tilde{x}(s)](x + l_D)\Big] \le \mathbb{E}\Big[\mathbb{1}[x > l_P^s](x + l_D)\Big] && \text{(IC for D)} \end{aligned}$$

where $\tilde{x}(s)$ is the induced cutoff in equilibrium by D's strategy $s$. The objective function implies that $s$ should be minimal given the fixed $\tilde{x}(s)$, thus P's IC constraint implies that $s = \tilde{x} - l_P^s$ or $\tilde{x} = s + l_P^s$.

The IC constraint for D requires that, for D to be willing to offer any settlement, she should expect to pay less with the settlement strategy (left-hand-side), than foregoing the attempt of offering a pre-litigation settlement entirely (right-hand-side).

D's problem therefore reduces to

$$\min_s \quad F(s + l_P^s)s + \int_{s+l_P^s}^{\overline{x}} (x + l_D)f(x)\mathrm{d}x = F(s + l_P^s)(s - l_D) + \int_{s+l_P^s}^{\overline{x}} xf(x)\mathrm{d}x + l_D \tag{B.1}$$

$$\text{s.t.} \quad F(s + l_P^s)s \le \int_s^{s+l_P^s} (x + l_D)f(x)\mathrm{d}x \tag{B.2}$$

Assuming (B.2) holds, the first-order and second-order conditions of (B.1) are

$$\frac{f(s^* + l_P^s)}{F(s^* + l_P^s)} = \frac{1}{l_D + l_P^s} \qquad \text{and} \qquad \frac{\partial}{\partial t}\left[\frac{f(t)}{F(t)}\right]\Bigg|_{t=s^*+l_P^s} \le 0.$$

**Proposition B.1.** *If $f(x)$ is log-concave, then the screening equilibrium $s^*$ of the pre-litigation model is unique, if it exists.*

*Proof.* If $f(x)$ is log-concave, then the ratio $\frac{f(t)}{F(t)}$ is monotone decreasing (Bagnoli and Bergstrom, 2004). Therefore the solution to the first-order condition is unique. The monotone decreasing property also satisfies (B.2), thus the equilibrium exists and is unique. □

**Proposition B.2.** *The pre-litigation settlement offered by D in the equilibrium, $s^*$, is*

- *non-decreasing if $l_D$ increases;*
- *ambiguous if $l_P$ increases; and*
- *unresponsive to changes in k.*

*Proof.* Calculating comparative statics using the first-order-condition, we have

$$\frac{\partial s^*}{\partial l_D} = -\frac{\frac{1}{(l_D+l_P^s)^2}}{\frac{\partial}{\partial t}\left[\frac{f(t)}{F(t)}\right]\Big|_{t=s^*+l_P^s}} \geq 0 \quad \text{by the second-order condition.}$$

$$\frac{\partial s^*}{\partial l_P} = \begin{cases} -\dfrac{0}{\frac{\partial}{\partial t}\left[\frac{f(t)}{F(t)}\right]\Big|_{t=s^*+k}} = 0 & \text{if} \quad l_P < k \\ -\dfrac{\frac{\partial}{\partial t}\left[\frac{f(t)}{F(t)}\right]\Big|_{t=s^*+l_P} + \frac{1}{(l_D+l_P)^2}}{\frac{\partial}{\partial t}\left[\frac{f(t)}{F(t)}\right]\Big|_{t=s^*+l_P}} = -\dfrac{1}{(l_D+l_P)^2\frac{\partial}{\partial t}\left[\frac{f(t)}{F(t)}\right]\Big|_{t=s^*+l_P}} - 1 & \text{if} \quad l_P \geq k \end{cases}$$

where sign is ambiguous if $l_P \geq k$.

Similarly,

$$\frac{\partial s^*}{\partial k} = \begin{cases} -\dfrac{\frac{\partial}{\partial t}\left[\frac{f(t)}{F(t)}\right]\Big|_{t=s^*+k} + \frac{1}{(l_D+l_P)^2}}{\frac{\partial}{\partial t}\left[\frac{f(t)}{F(t)}\right]\Big|_{t=s^*+k}} = -\dfrac{1}{(l_D+l_P)^2\frac{\partial}{\partial t}\left[\frac{f(t)}{F(t)}\right]\Big|_{t=s^*+k}} - 1 & \text{if} \quad l_P \leq k \\ -\dfrac{0}{\frac{\partial}{\partial t}\left[\frac{f(t)}{F(t)}\right]\Big|_{t=s^*+l_P}} = 0 & \text{if} \quad l_P > k \end{cases}$$

where sign is ambiguous if $l_P \leq k$. □

**Proposition B.3.** *The equilibrium cutoff to file suit $\tilde{x}^* = s^* + l_P^s$ and the average pre-trial settlement are,*

1. *if $l_P < k$,*
   - *non-decreasing if $l_D$ increases;*
   - *unresponsive if $l_P$ increases;*
   - *non-decreasing if k increases.*
2. *if $l_P > k$,*
   - *non-decreasing if $l_D$ increases;*
   - *non-decreasing if $l_P$ increases;*
   - *unresponsive if k increases.*

*Proof.* Because $\tilde{x}^* = s^* + l_P^s$, we have

$$\frac{\partial \tilde{x}^*}{\partial l_D} = \frac{\partial s^*}{\partial l_D} \geq 0 \quad \text{by the proof of Proposition B.2}$$

$$\frac{\partial \tilde{x}^*}{\partial l_P} = \frac{\partial s^*}{\partial l_P} + \frac{\partial l_P^s}{\partial l_P} = \begin{cases} -\frac{0}{\frac{\partial}{\partial t}\left[\frac{f(t)}{F(t)}\right]\Big|_{t=s^*+k}} = 0 & \text{if} \quad l_P < k \\ -\frac{\frac{\partial}{\partial t}\left[\frac{f(t)}{F(t)}\right]\Big|_{t=s^*+l_P} + \frac{1}{(l_D+l_P)^2}}{\frac{\partial}{\partial t}\left[\frac{f(t)}{F(t)}\right]\Big|_{t=s^*+l_P}} + 1 = -\frac{1}{(l_D+l_P)^2\frac{\partial}{\partial t}\left[\frac{f(t)}{F(t)}\right]\Big|_{t=s^*+l_P}} \geq 0 & \text{if} \quad l_P \geq k \end{cases}$$

Similarly,

$$\frac{\partial \tilde{x}^*}{\partial k} = \frac{\partial s^*}{\partial k} + \frac{\partial l_P^s}{\partial k} = \begin{cases} -\frac{\frac{\partial}{\partial t}\left[\frac{f(t)}{F(t)}\right]\Big|_{t=s^*+k} + \frac{1}{(l_D+l_P)^2}}{\frac{\partial}{\partial t}\left[\frac{f(t)}{F(t)}\right]\Big|_{t=s^*+k}} + 1 = -\frac{1}{(l_D+l_P)^2\frac{\partial}{\partial t}\left[\frac{f(t)}{F(t)}\right]\Big|_{t=s^*+k}} \geq 0 & \text{if} \quad l_P \leq k \\ -\frac{0}{\frac{\partial}{\partial t}\left[\frac{f(t)}{F(t)}\right]\Big|_{t=s^*+l_P}} = 0 & \text{if} \quad l_P > k \end{cases}$$

The results above establishes the statement regarding $\tilde{x}^*$. The statement about average pre-trial settlement $\int_{\tilde{x}^*}^{\overline{x}} x f(x|x > \tilde{x}^*)\mathrm{d}x$ then follows from Lemma 7.

□

These results are in sharp contrast to those in our signaling model, both when $l_P < k$ (Proposition 10), and when $l_P > k$ (Proposition 12). In particular, the very different predictions on the effects of $l_P$ and $k$ may lend to empirical opportunities to distinguish the two theories in the data.

# C Alternative version - legal costs increasing in judgment value $x$

An extension that renders the modeling assumptions more realistic is for legal costs to vary with the underlying case value $x$. More complicated cases take more resources to prepare the complaint, to respond, to discover, and to go through trial. It is therefore worthwhile to investigate if the key comparative statics results of the model are robust to such an extension.

In this extension, let function $c_P(l_P, x)$ $c_D(l_D, x)$ and $c_k(k, x)$ be the legal cost functions for P and D's cost to file suit, and the trial cost, respectively. In particular, $c_D(l_D, x)$ may also be interpreted as the cost D has to incur in order to learn the true value of $x$. We assume all these legal cost functions satisfy the following conditions—(i) increasing in their the cost parameter $l_P, l_D, k$ for all $x$, respectively; (ii) continuous, increasing and concave in $x$; (iii) strictly positive for completely trivial case $x = 0$. A special example is for $c_P(l_P, x) = l_P$, which reduces to the model in the article, similarly for $c_D(l_D, x)$ and $c_k(k, x)$. Concavity in assumption (ii) ensures that higher value cases are more worthwhile to go through the legal system, i.e. $x - c_P(l_P, x)$ is increasing in $x$, etc. Assumption (iii) maintains the inclusion of "non-credible" suits that, under complete information case, would not have been settled.

The set of law-suit credible cases is $L_P^s = \{x : x > \max(c_P(l_P, x), c_k(k, x))\}$. Define the law-suit credibility threshold as $l_P^s = \{x : x = \max(c_P(l_P, x), c_k(k, x))\}$.

**Lemma C.1.** *Under the stated assumptions (i) through (iii), for fixed cost parameters $l_P, l_D$ and $k$, the law suit-credibility threshold, $l_P^s$, is unique, which implies that the set of law-suit-credible cases is convex.*

*Proof.* For fixed cost parameters, define shorthand $f_P(x) = c_P(l_P, x)$, and similarly $f_k(x) = c_k(k, x)$.

If either $f_P$ or $f_k$ dominates another for all $x$, then the result follows obviously because the cost functions are concave and above zero at $x = 0$. It suffices to consider the case where they cross at some $x_c$. Without loss of generality, assume $f_P > f_k$ for all $x < x_c$ and vice versa.

Suppose $l_P^s$ is not unique, i.e. $x$ crosses $\max(c_P(l_P, x), c_k(k, x))$ at least twice, denote two of the intersections $x_1$ and $x_2$, where $x_1 < x_2$. Then it must be that $x_1 \leq x_c$ and $x_2 > x_c$, which implies $x_1 = f_P(x_1)$ and $x_2 = f_k(x_2)$. Because $f_P > f_k$ for all $x < x_c$, $f_P(x_1) \geq f_k(x_1)$. Moreover, because $f_k(0) > 0$, there must exist a intersection between $f_k$ and $x$ in $(0, x_1]$. But $f_k(x_2) = x_2 > 0$ implies $f_k$ intersects with $x$ twice, a contradiction to $f_k(0) > 0$ and the concavity of $f_k$. □

By Lemma C.1, the structure of D's willingness to pay for demand in the article is well defined. The equilibrium necessary condition (3) generalizes to

$$\tilde{X} \equiv \left\{ l_P^s \leq \tilde{x} \leq \overline{x} : \int_{l_P^s}^{\tilde{x}} (x + c_D(l_D, x)) f(x | x \leq \tilde{x}) \mathrm{d}x \geq \tilde{x} - l_P(l_P, x) \right\}. \tag{C.1}$$

Given this condition, a similar proof to Proposition 4 shows the same insight—the maximums and minimums of each convex subset of $\tilde{X}$ constitutes all the equilibria of the game. Moreover, the proof to Proposition 6 follows through as is, and shows that the equilibrium with substantive demand that satisfies the intuitive criterion is uniquely defined as the greatest element of $\tilde{X}$.

It remains to check if the comparative statics results remain robust in this extension. The previous condition of whether $l_P > k$ now generalizes to if P's law-suit credibility threshold $l_P^s$ is determined by trial cost $c_k(k, x)$ or filing cost $c_P(l_P, x)$. Specifically, when $x$ crosses the upper envelop of these two cost functions, which one comes on top—whether $c_P(l_P, l_P^s) > c_k(k, l_P^s)$ or otherwise. The following propositions are analogous to Propositions

9 through 12. They show that the empirical predictions summarized in Table 1 and 2 are fully generalized in this extension.

**Proposition C.1.** *If P's law-suit credibility threshold $l_P^s$ is determined by trial cost $c_k(k,x)$, then in the equilibrium, $\tilde{x}^*$ is non-decreasing if*

- *$l_D$ increases;*
- *$l_P$ increases; or*
- *$k$ decreases.*

*Proof.* The proof uses the same reasoning of that of Proposition 9. One sufficient condition to evaluate the effect for parameters on $\tilde{x}^*$ is to evaluate their effects on the defining function of $\tilde{X}$. Therefore, based on (C.1), we define

$$\Omega = \tilde{x} - l_P(l_P, x) - \int_{l_P^s}^{\tilde{x}} (x + c_D(l_D, x)) f(x|x \leq \tilde{x}) \mathrm{d}x.$$

And then $\frac{\partial \tilde{x}^*}{\partial c} \geq 0$ if $\frac{\partial \Omega}{\partial c} \leq 0$.

Taking derivatives of $\Omega$ with respect to the cost parameters $l_P, l_D, k$, we have the results.

$$\frac{\partial \Omega}{\partial l_P} = -\underbrace{\frac{\partial c_P(l_P, \tilde{x})}{\partial l_P}}_{\geq 0 \text{ by assump. (i)}} + \frac{1}{F(\tilde{x})}[\underbrace{l_P^s + c_D(l_D, l_P^s)]f(l_P^s)}_{\geq 0} \cdot \frac{\partial l_P^s}{\partial l_P}] = \begin{cases} \lesseqgtr 0 & \text{if } \frac{\partial l_P^s}{\partial l_P} > 0 \\ \leq 0 & \text{if } \frac{\partial l_P^s}{\partial l_P} = 0 \end{cases}.$$

$$\frac{\partial \Omega}{\partial l_D} = -\underbrace{\frac{1}{F(\tilde{x})} \int_{l_P^s}^{\tilde{x}} \frac{\partial c_D(l_D, x)}{\partial l_D} f(x|x \leq \tilde{x}) \mathrm{d}x}_{\geq 0 \text{ since } \frac{\partial c_D(l_D, x)}{\partial l_D} \geq 0 \forall x \text{ by asssumption (i)}} \leq 0.$$

$$\frac{\partial \Omega}{\partial k} = \frac{1}{F(\tilde{x})}[\underbrace{l_P^s + c_D(l_D, l_P^s)]f(l_P^s)}_{\geq 0} \cdot \frac{\partial l_P^s}{\partial k}] = \begin{cases} \geq 0 & \text{if } \frac{\partial l_P^s}{\partial k} \geq 0 \\ = 0 & \text{if } \frac{\partial l_P^s}{\partial k} = 0 \end{cases}.$$

□

**Proposition C.2.** *If P's law-suit credibility threshold $l_P^s$ is determined by trial cost $c_k(k,x)$, then in the equilibrium, the pre-litigation settlement amount $m(\tilde{x}^*)$, and the average pre-trial settlement amount, $\int_{\tilde{x}^*}^{\overline{X}} x f(x|x > \tilde{x}^*) dx$, are non-decreasing if*

- *$l_D$ increases;*
- *$l_P$ increases; or*
- *$k$ decreases.*

*Proof.* Follows from Proposition C.1 and the proof of Proposition 10. □

Under the alternative scenario where filing cost dictates the law suit-credibility threshold, the comparative statics results are summarized below.

**Proposition C.3.** *If P's law-suit credibility threshold $l_P^s$ is determined by filing cost $c_P(l_P, x)$, then in the equilibrium, $\tilde{x}^*$ is*

- *non-decreasing if $l_D$ increases;*
- *ambiguous if $l_P$ increases; and*
- *unresponsive to changes in k.*

*Proof.* Implied by the proof of Proposition C.1. □

**Proposition C.4.** *If P's law-suit credibility threshold $l_P^s$ is determined by filing cost $c_P(l_P, x)$, then in the equilibrium, the pre-litigation settlement amount $m(\tilde{x}^*)$, and the observed average pre-trial settlement amount, $\int_{\tilde{x}^*}^{\overline{x}} xf(x|x > \tilde{x}^*)dx$,*

- *are non-decreasing if $l_D$ increases;*
- *are ambiguous if $l_P$ increases; and*
- *does not respond to changes in k.*

*Proof.* Omitted for similarity to that of Proposition C.2. □

Finally, the proofs for Propositions 14 and 16 go through as is for this extension. Therefore we conclude with the following remark.

**Remark.** Given assumptions (i)-(iii) on the cost functions, the entire set of empirical predictions of the model remains robust to the extension of variable legal cost.